Financial Cycles with Heterogeneous Intermediaries

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Abstract

We develop a dynamic macroeconomic model with heterogeneous financial intermediaries and endogenous entry. Time-varying endogenous macroeconomic risk arises from the risk-shifting behaviour of the cross-section of financial intermediaries. When interest rates are high, a decrease in interest rates stimulates investment and decreases aggregate risk. In contrast, when they are low, further stimulus can increase financial instability while inducing a fall in the risk premium. In this case, there is a trade-off between stimulating the economy and financial stability. This provides a model of the risk-taking channel of monetary policy.

JEL Codes: E32, E44, E52, G21.

Keywords: Financial Cycle, Risk-taking channel of monetary policy, Leverage, Systemic Risk.

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1 Introduction

The global financial crisis of 2007-2008 has called into question our modelling of the role of financial intermediaries in the economy. The financial sector, far from being a veil, plays a key role in the transmission and amplification of shocks and in driving fluctuations in aggregate risk. The precise mechanisms by which this happens are still debated. In particular, understanding the underlying forces driving endogenous systemic risk, the concentration of risk in some balance sheets and the interactions between monetary policy and financial stability are key issues. A long tradition of scholars such as Fisher (1933), Minsky (1977) and Kindleberger (1978) argued that financial sector expansions and contractions are important drivers of fluctuations in economic activity and financial stability. Kaminsky and Reinhart (1999) and Reinhart and Rogoff (2009*b*) among others show that financial crises tend to be preceded by a rapid expansion of debt. Schularick and Taylor (2012) study the long run dynamics of money, credit and output over the period 1870-2008 and find that financial crises tend to be "booms gone bust". Our paper is distinct from the recent macro-finance literature in two main ways: we focus on the endogenous dynamics of the *boom phase* of the financial cycle and we show the importance of *heterogeneity in risk-taking* to explain the boom, systemic risk build ups and fluctuations in risk premia. In addition, our model features a risk-taking channel of monetary policy.

Financial cycles have been analysed in the literature typically through the lenses of models featuring one representative financial intermediary subject to capital market frictions. In contrast, we emphasise the importance of heterogeneity in risk taking across financial intermediaries in driving aggregate outcomes. Increases in the market shares of the most risk taking intermediaries concentrate risk on large balance sheets and play a key role in the build-up of financial fragility. For Sweden, Englund (2016) explains how between 1985 and 1990 the rate of increase of lending by financial institutions jumped to 16% due in part to deregulation with rapid shifts in market shares. There was a significant correlation between the rate of credit expansion of specific institutions and their subsequent credit losses in the crisis, leading to bailouts. For Spain, Santos (2017) emphasizes how between 2002 and 2009, the regional banks (cajas) leveraged a lot to invest in the real estate sector, with their combined balance sheet reaching 40% of Spanish GDP in 2009. Some (Bancaja) more than tripled their balance sheet while more "conservative" ones (Catalunya Caixa) doubled it. They ended up all being nationalized in the crisis. In Germany, as described by Hellwig (2018), Landesbanken and local savings banks whose borrowing was guaranteed by German Lander and municipalities until 2005 took the opportunity to gorge on cheap funds increasing their debt by around €250bn over the period 2001 to 2005. Deutsche Bank leveraged up to quadruple the size of its balance sheet from about €0.5 trillion in early 1990s to about €2 trillion in 2008 as a RoE of 25% was regularly targeted by the bank CEO. German taxpayers ended up paying about €70 billion to support their financial institutions. In the US, Wilmarth (2013) mentions the high risk culture of the too-big-to-fail Citigroup as a possible explanation behind the massive expansion of its balance sheet during the boom years. Citigroup nearly doubled the share of its subprime mortgage business from 10% in 2005 to 19% in 2007. During the period 2007 to the spring of 2010, Citigroup recorded more than $130 billion in credit losses and write-downs. It received its first government bailout in October 2008 (it was bailed out 3 times in total). Accounting for such heterogeneity in risk taking behaviour is important as assets concentrate in risky balance sheets and this has systemic risk implications.

A large literature has recognized the centrality of financial frictions such as costly state verification, collateral, net worth or Value-at-Risk (VaR) constraints for representative firms and intermediaries[1]. But that literature has mostly focused on the transmission and amplification of shocks rather than the endogenous risk build-up phase[2]. And it has ignored heterogeneity in risk taking and the concentration of risk in some balance sheets. We build a novel framework with a continuum of financial intermediaries heterogeneous in their VaR constraints[3] and a moral hazard friction due

[1]See in particular, but not only, Bernanke and Gertler (1989), Kiyotaki and Moore (1997)), Lorenzoni (2008), Mendoza (2010), Adrian and Shin (2010), Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013).

[2]In Kiyotaki and Moore (1997), the interaction between endogenous credit limits and asset prices generates a powerful transmission mechanism for productivity shocks while Guerrieri and Lorenzoni (2017) presents a Bewley model with borrowing limits and credit crunch shocks.

[3]Adrian and Shin (2014) provides microfoundations for VaR constraints. Stulz (2016) discusses the importance of Value-at-Risk constraints in the risk management practices of financial intermediaries.

to limited liability and government guarantees which generate risk-shifting[4]. Heterogeneous VaRs may reflect different risk attitudes by the boards of financial intermediaries or different implementations of regulatory constraints across institutions and supervisors. In our model, the dynamics of the *distribution of leverage* across intermediaries is a key determinant of financial stability. When high risk-taking intermediaries are dominant, they increase the price of risky assets and concentrate most of the aggregate risk on their balance sheets. The leverage distribution across intermediaries becomes very positively skewed and financial fragility increases during the boom phase, as a large fraction of assets are in the hands of intermediaries with a higher probability of default[5]. This tends to happen when financing costs are low, which may be due to deregulation, a savings glut, expansionary monetary policy or when volatility is low. The cyclical dynamics of the leverage distribution are consistent with the evidence. In contrast we do not find in the panel data any heterogeneous dynamics of leverage linked to net worth.

In our model, credit booms generated by low costs of funds are associated with worsened financial stability and lower risk premia: these are "*bad booms*" in the terminology of Gorton and Ordoñez (2019). This is consistent with the evidence reported in Krishnamurthy and Muir (2017) that spreads tend to be low before crises.[6] Our model also accounts for the *volatility paradox*: periods where systemic risk is high even though the volatility of the fundamentals is low. We provide therefore a complementary view of financial fragility from Gennaioli, Shleifer and Vishny (2012). For these authors, excess risk taking comes from neglecting some improbable risk. In our model, it is the presence of limited liability that leads bankers to optimally ignore downside risk within the default region, while government guarantees insure depositors[7]. Our framework

[4]Allen and Gale (2000) have shown that current and future credit expansions can increase risk-shifting and create bubbles in asset markets. Nuño and Thomas (2017) show that the presence of risk-shifting creates a link between asset prices and bank leverage.

[5]Default is costly and the cost of default is proportional to the balance sheet size of defaulting intermediaries (see Section 6).

[6]These authors note that in contrast standard models such as He and Krishnamurthy (2013) or Brunnermeier and Sannikov (2014) "will not match the pre-crisis spread evidence. In these models, a prolonged period in which fragility and leverage rises will also be coupled with an increase in spreads and risk premia. That is, the logic of these models is that asset prices are forward looking and will reflect the increased risk of a crisis as fragility grows".

[7]Baron and Xiong (2017) show that creditors of banks do not price the risk taken by bankers during credit expansions. Deposit guarantees have also the effect of ruling out bank runs in our framework.

also generates "*good booms*", driven by high expected productivity, which increase investment, do not increase financial instability and have a very limited effect on the risk premium.

Related Literature. A few papers have, like us, put their emphasis on the boom phase of the financial cycle. Lorenzoni (2008) shows that credit booms can be inefficient due to a pecuniary externality working through asset prices. In Martinez-Miera and Suarez (2014), bankers determine their exposure to systemic shocks by trading-off the risk-shifting gains due to limited liability with the value of preserving their capital after a crisis. Malherbe (2015) and Gersbach and Rochet (2017) present models with excessive credit during economic booms as increased lending by an individual bank exerts a negative externality on all other banks. A small set of papers have analysed financial sectors with heterogeneous agents. Boissay, Collard and Smets (2016) features intermediaries that are heterogeneous in their intermediation skills. In Guerrieri and Uhlig (2016), due to adverse selection, worse borrowers take loans when costs of funds are low, which may induce a crash of the credit market. Bolton, Santos and Scheinkman (2016) show the existence of a "cream skimming externality" whereby opaque OTC exchanges' remove good projects from organized exchanges, potentially leading to oversized financial markets and higher revenues for informed dealers. Martinez-Miera and Repullo (2017) analyse "search for yield" in an environment where riskier entrepreneurs endogenously borrow from monitoring banks while safer entrepreneurs borrow from shadow banks. Begenau and Landvoigt (2021) study the role of intermediaries as liquidity providers. The heterogeneity (banks versus shadow banks) they focus on comes from regulatory differences: like us, their intermediaries have limited liability and their banks have a deposit guarantee which leads to moral hazard. Unlike us, the regulatory constraint imposed on banks has the form of an equity capital requirement and shadow banks may face bank runs. They perform a rich normative analysis of the relative sizes of banks and shadow banks, where costly default and provision of liquidity are traded off against one another. In contrast we focus on heterogeneity in leverage and risk taking (including across banks) and its effect on financial stability as funding costs change. We show that the distribution of leverage across banks is tightly related to systemic risk and that

For recent models focusing on runs see Gertler and Kiyotaki (2015) and Angeloni and Faia (2013)).

monetary policy is a powerful driver of risk premia, as in the data. A series of important papers, Geanakoplos (2003), Geanakoplos (2010), Fostel and Geanakoplos (2008) study leverage cycles driven by wealth reallocations between optimists and pessimists[8]. On the empirical side, our work relates to a recent paper by Koijen and Yogo (2019) who test models where heterogeneity across institutional investors is an important driver of asset pricing and to Coimbra, Kim and Rey (2021) who build a measure of systemic risk from structural estimates of the entire cross-section of Value-at-Risk parameters of intermediaries. Our model attempts to perform in macro-finance something similar to what Melitz (2003) has done in international trade by relating aggregate outcomes to underlying microeconomic heterogeneity. We are not aware of any other paper in the macro-finance literature that pursues a similar aim.

Risk Taking Channel of Monetary Policy. In our model, there is an endogenous non-linearity in the trade-off between monetary policy, which affects the funding costs of intermediaries, and financial stability[9]. When the level of interest rates is high, a fall in interest rates leads to an increase in leverage (*intensive margin*) and to entry of less risk-taking intermediaries into the market for risky projects (*extensive margin*). The average intermediary is then less risky, so a fall in the cost of funds improves financial stability and expands the capital stock: there is no trade-off between stimulating the economy and financial stability. However, when interest rates are very low, a further decrease benefits the most leveraged risk-taking intermediaries and risk concentration increases. Additionally, when the aggregate capital is stock is inelastic, competition may drive out the more prudent players. Stimulating the economy shifts the distribution of assets towards the more risk-taking intermediaries, which have a higher default risk and increases aggregate risk-shifting. This non-monotonicity constitutes a

[8]Korinek and Nowak (2017) use an evolutionary dynamics approach to characterize the distribution of bankers' wealth: good shocks raise the fraction of wealth controlled by high risk takers. Kekre and Lenel (2020) emphasize the heterogeneity in household's marginal propensity to take risk to explain the stock market response to monetary policy shocks while Kaplan, Mitman and Violante (2017) focus on household heterogeneity and beliefs to explain housing booms.

[9]Our model is about the behaviour of the real interest rate, so the connection with monetary policy is only partial. Any change in regulation that affects funding costs would have similar implications. So would higher savings rates or large capital inflows. An extension of the model featuring nominal variables is left for future work.

substantial difference from the existing literature and is a robust mechanism coming from heterogeneity in risk taking and the general equilibrium feedback effect of asset returns. It provides a novel way to model the risk-taking channel of monetary policy analysed in Borio and Zhu (2012), Challe, Mojon and Ragot (2013)[10], Angeloni, Faia and Lo Duca (2015) and Bruno and Shin (2015)[11] and the well documented effect of monetary policy on asset prices (see e.g. Miranda-Agrippino and Rey (2020)).

The paper is organized as follows. Section 2 provides stylized facts on the heterogeneous behaviour of leverage in the cross-section of intermediaries. Section 3 describes the model and introduces measures of systemic risk. Section 4 presents the main results in partial equilibrium to build intuition. Section 5 shows the general equilibrium results and responses to monetary policy and productivity shocks. We also discuss *good booms* and *bad booms* and the risk taking channel of monetary policy. The case of financial crises with costly intermediary default is analyzed in section 6. Section 7 concludes.

2 Stylized facts on the cross-section of intermediary balance sheets

In this section we present some stylized facts on the cyclical properties of the cross-section of financial intermediary balance sheets. We use balance sheet data from Bankscope (see Appendix D) to compute leverage at the intermediary level. Leverage is defined as the ratio of assets over equity at book value, a definition that will be kept for the theoretical model of the following sections.

In Figure 1 we show the time series of asset-weighted leverage for different quantiles

[10]Challe, Mojon and Ragot (2013) describe a two-period model with heterogeneous intermediaries and limited liability which, like ours, features a link between interest rates and systemic risk. They focus on portfolio choice and heterogeneity in equity of intermediaries while we emphasize aggregate uncertainty and differences in risk taking. Unlike them, we embed the financial sector in a DSGE model.

[11]Recent empirical evidence on the risk-taking channel of monetary policy has been provided by Dell'Ariccia, Laeven and Suarez (2017) on US data, Jimenez et al. (2014) and Morais et al. (2019), exploiting credit registry data on millions of loans of the Spanish and Mexican Central Banks. Using detailed Turkish data, Baskaya et al. (2017) highlight the importance of bank heterogeneity for credit creation and the transmission of global financing cost shocks. Coimbra and Rey (2018) show that in a cross section of countries, credit creation tends to be more elastic to decline in funding costs when the leverage distribution of the banking system is concentrated.

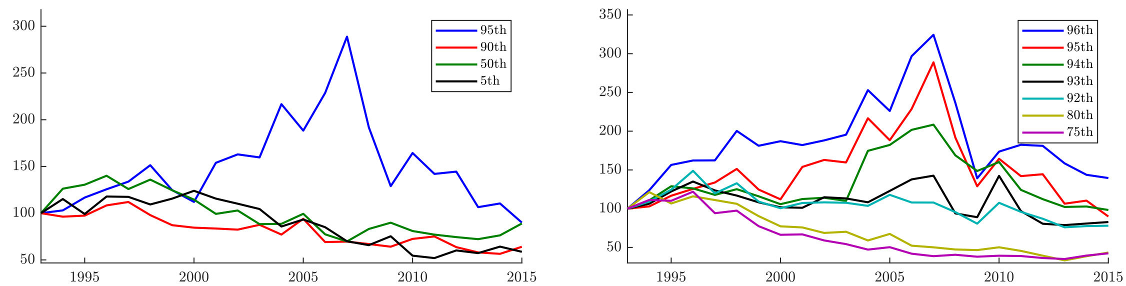

Figure 1: Evolution of asset-weighted leverage by quantiles (base year=1993)
Quantiles of each year's cross-sectional distribution of asset-weighted leverage, rebased to 100 at the beginning of sample. Asset-weights calculated by dividing balance sheet size by the respective yearly mean balance sheet size.

of the leverage distribution, namely the top 5% (blue), top 10% (red), median (green) and bottom 5% (black) on the left panel and for a denser set of quantiles above the 75th quantile on the right panel. Values were rebased to 100 at the beginning of the sample in 1993, to highlight the stark differences in dynamics and were weighted by assets to give a meaningful relevance to larger institutions. In the years that preceded the financial crisis there was a strong increase in leverage in the top quantiles (for example for the top 5%, leverage is multiplied by 2.5 between 2000 and 2008) but not in lower quantiles. There was a negative pre-crisis correlation between the top quantiles and both the median and the bottom quantiles, while the correlation is positive between the median and the bottom quantiles. The quantiles are weighted by balance sheet size so it was the large, highly levered intermediaries that were increasing leverage the most during the pre-crisis period. The behaviour across quantiles is quite smooth as evidenced by the right panel of Figure 1. We show in Figure A1 of the Appendix the equivalent graphs for the dynamics of leverage across equity quantiles. In contrast to leverage, there are no discernible patterns.

In Figure 2, using binned scatter plots we show the mean of leverage within each asset decile (left) and equity decile (right) for the pre-crisis sample (each dot represents an average of 41 distinct financial intermediaries). Leverage is higher in a convex fashion for the larger deciles of assets. In contrast, there is no clear relationship between leverage and equity deciles. To illustrate the dynamics of the convex relationship between leverage and asset size, in Figure A2 we show binned scatter plots of leverage

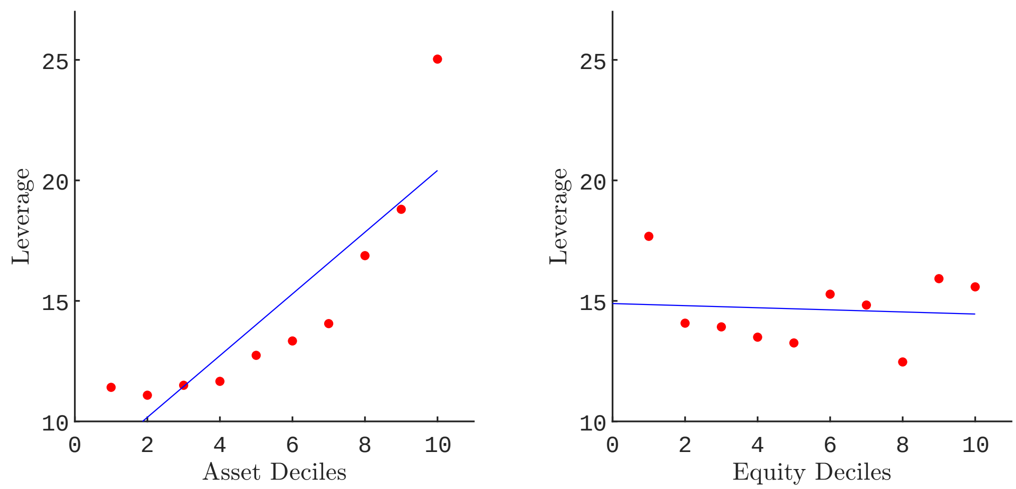

Figure 2: Mean leverage by asset deciles (left) and equity deciles (right).
Binned scatter plots with intermediaries grouped in the x-axis into 10 bins by balance sheet size (left panel) and equity (right panel). Red dots represent the mean leverage (unweighted) of each group. Each bin contains an average of 41 distinct financial intermediaries and 606 observations.

as a function of asset quantiles by year. Each bin contains about 30 intermediaries and we plot the median leverage for each bin. We also plot in red a cubic fit line. Leverage is increasing in asset quantiles, meaning that the larger intermediaries also tend to be the most levered for each year. Strikingly, the convexity of leverage with respect to size increases significantly in the pre-crisis period to culminate around 2007. This period (the "boom" part of the financial cycle) is characterized by low costs of funds due to abundant liquidity and light touch regulation. After the crisis and an increase in regulation following Basel 3, the convexity abates with a slight increase at the end of the sample. Hence leverage is not only largest for the bigger intermediaries, but it is also more reactive over the financial cycle.

Stylized fact 1: *Leverage dynamics are highly heterogeneous across intermediaries. Leverage is a convex function of balance sheet size. In contrast, there is no clear relation between leverage and equity quantiles. The sensitivity of leverage to the cycle is higher for larger, more leveraged intermediaries.*

These properties of convexity and sensitivity translate into cyclical movements in the concentration of assets. To illustrate this, we compute the share of total assets held by the top 5% most levered intermediaries. Figure 3 is a scatter-plot of that share (top 5%) on the vertical axis with the Effective Fed Funds Rate both in real terms (left panel) and in nominal terms (right panel) on the horizontal axis. Each point is a yearly observation. There is a negative correlation between the Effective Fed Funds Rate and the top 5% share. There are some outliers, in particular the three points which are above the regression lines (share of top 5% above 65% of total assets despite relatively high real or nominal rates). They correspond to the years 2006, 2007 and 2008, immediately before the crisis. One possible interpretation is that the real cost of funds in those years declined more than the Fed Funds rate proxy would suggest due to the substantial increase in the use of very short term repo markets (overnight repo) used by large banks and the lax regulatory framework.[12]

Stylized fact 2: *There is a negative correlation between the share of assets of the top 5% most levered intermediaries and the cost of funds as proxied by the real or the nominal effective Fed Funds rate. Concentration of assets increases as costs of funds go down.*

The evidence presented above indicates that here is a strong heterogeneity within the financial sector in terms of correlation of leverage and interest rate over time. Up to 2007, the correlation between the top 5% and the real effective Fed Funds rate is -0.34 but it is positive for the median (0.58) and bottom 5% (0.41).[13] To investigate this further, we look at the relationship between individual bank leverage and interest rates, allowing for different responses across the distribution. We run the following baseline panel regression:

$$Lev_{i,t} = \beta_0 + \beta_1 Lev_{i,t-1} + \beta_2 FF_t + \beta_3 Top5_{i,t} + \beta_4 FF_t \times Top5_{i,t} + \alpha_i + \varepsilon_{i,t} \quad (1)$$

[12] Banks in the top 5% of the leverage distribution in 2007 include Barclays, Bear Stearns, Citigroup, Corporate One Federal Credit Union, Deutsche Bank, among others. The top 5% leverage quantile (weighted by assets) include Ageas/Fortis, Bank of America, Barclays, Citigroup, Credit Agricole, Deutsche Bank, JP Morgan.

[13] Results are qualitatively very similar if we use instead nominal rates. The correlation between the top 5% and the nominal effective Fed Funds rate is -0.20 and again positive for the median (0.49) and bottom 5% (0.24). Results also similar if we use only US banks.

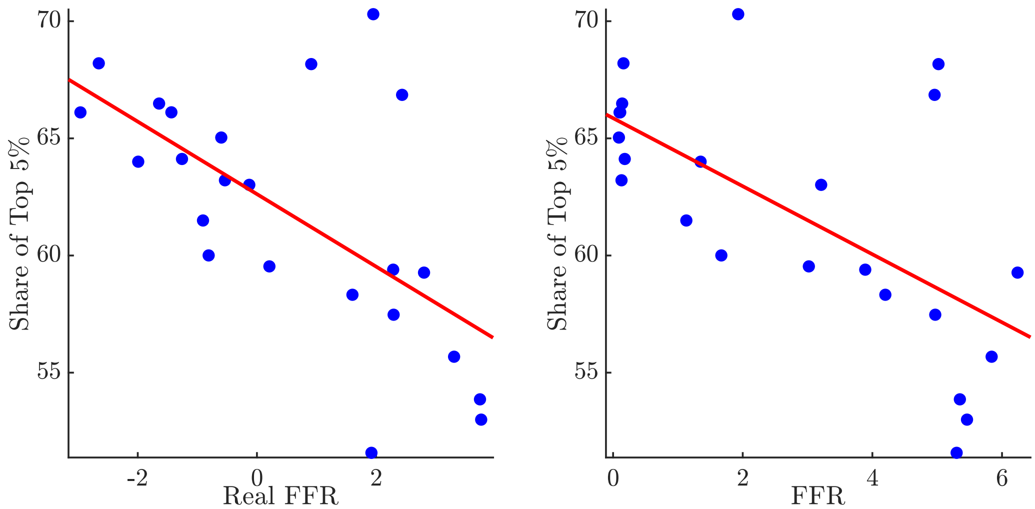

Figure 3: Share of assets of the top 5% most levered intermediaries in total intermediaries' assets and the real Effective Fed Funds Rate (left panel) and the nominal effective Fed Funds rate (right panel) in pp.

$Top5$ is a dummy variable that takes the value 1 if the intermediary is in the top 5% of intermediaries by leverage. FF is the real Fed Funds Rate. β_1 measures persistence of leverage and β_2 the response of leverage to interest rates. β_3 picks up the average difference in leverage in the two groups. Our object of interest is β_4, which captures the heterogeneity of the response of leverage to interest rates for the top 5%. Results can be seen in Table A1. The first column is the baseline specification of Equation (1). All regressions have financial intermediary fixed effects. As expected, there is significant persistence in leverage as β_1 is positive and highly significant. Also, leverage is significantly larger on average for the $Top5$ group. The coefficient of the real Fed Funds Rate is not significant when $Top5 = 0$ but is highly significant and negative when $Top5 = 1$. That is, leverage and interest rates are negatively correlated for highly leveraged intermediaries, but this is not the case for other intermediaries. The following columns show the robustness of this relationship. In the second column we show that introducing time fixed effects does not affect our results. In the third column, we generate the dummy variable using asset-weighted leverage quantiles and in the fourth and fifth ones we generate it using the top 10% of, respectively, unweighted and asset-weighted leverage. Results hold qualitatively in all, although for the top 10% the point estimates are lower than their top 5% counterparts. We also interact FF with lagged leverage and again we find that the more levered an intermediary is, the more sensitive is its leverage to changes in the real Fed Funds Rate. In the final column, we conduct the baseline regression in first differences, with again similar results. We also run our baseline regression (with time fixed effects) using percentiles different from the top 5 and 10%. We show in Figure 4 a striking decay in the estimates, where the lower percentiles are the most leveraged intermediaries. This illustrates again that the more levered an intermediary is, the more sensitive its leverage is to movements in the real Fed Funds rate. The decay is present irrespective of using unweighted leverage quantiles (left panel) or asset-weighted ones (middle panel). In Table A2, we conduct

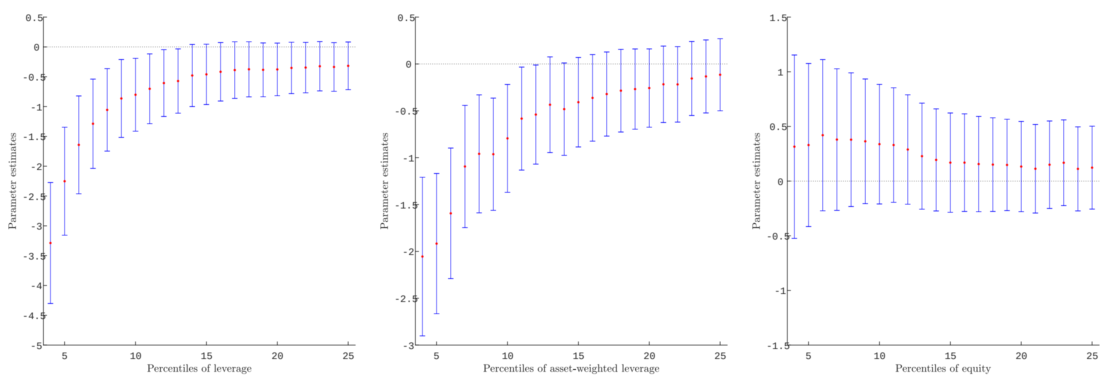

Figure 4: Interaction coefficients under different percentile choices
Point-estimates and 95% significance bounds for θ_3 in the following regression: $Lev_{i,t} = \theta_0 + \theta_1 Lev_{i,t-1} + \theta_2 TopX^j_{i,t} +$ $+ \theta_3 FF_t \times TopX^j_{i,t} + \nu_i + \mu_t + \varepsilon_{i,t}$. $TopX^j_{i,t}$ is a dummy variable, that takes the value 1 if the intermediary has leverage (left graph, $j = \{\}$), asset-weighted leverage (middle graph, $j = \{AW\}$) or equity (right graph, $j = \{Eq\}$) *above* the X-axis percentile value for year t.

the same type of exercises using equity. In contrast to leverage, no robust relationship emerges between equity and the sensitivity of leverage to interest rates. Additionally, we perform a "horse race" regression where we include both equity and leverage and find that only the interaction coefficient of leverage is significant. In the last column we also show that the interaction coefficient of leverage and balance sheet size is also not significant in a similar regression. The right panel of Figure 4 presents the estimates of the interaction term (point estimate and confidence interval) between the Fed Funds rate and equity percentiles. None is significant. The lack of relationship between equity and the elasticity of leverage to the Fed Funds rate does not depend on the percentile chosen.

Stylized fact 3: *There is a strong positive correlation between leverage and its elasticity with respect to the Fed Funds rate. Highly levered intermediaries are more sensitive to changes in the Fed Funds rate compared to less levered ones. In contrast, no clear relationship exists for equity quantiles.*

These stylized facts highlight the presence of a convex relationship between leverage and size, of a strong and specific heterogeneity in the dynamics of the cross-section of leverage and of heterogeneity in the sensitivity with respect to the Fed Funds rate. In contrast, there is no clear link between equity quantiles and leverage, nor any robust relation between heterogeneous equity levels and sensitivity to the Fed Funds rate. In the next section we present a model that is able to rationalize these new facts and explore their implications. Heterogeneity in leverage is a first order determinant of the dynamics of aggregate risk and of the macroeconomy in the model.

3 The Model

The general equilibrium model is composed of a representative risk-averse household who faces an intertemporal consumption saving decision, a continuum of risk-neutral, heterogeneous financial intermediaries, and a stylized central bank and government. There is aggregate uncertainty, in the form of productivity and monetary policy shocks. Given the heterogeneity in bank balance sheets, this will lead to heterogeneity in default risk in the intermediation sector. We introduce costly default in Section 6.

3.1 Households and the production sector

The representative household has an infinite horizon and consumes a final good C_t^H. She finances her purchases using labour income W_t and returns from a savings portfolio. We assume that the household has a fixed labour supply and does not invest directly in the capital stock K_t.[14] She can either save using a one-to-one storage technology S_t^H and/or deposit D_t^H with financial intermediaries at interest rate r_t^D. The return on deposits $R_t^D \equiv 1 + r_t^D$ is risk-free and guaranteed by the government. Intermediaries use deposits, along with inside equity ω_{it}, to invest in capital and storage. In Section 5 we will introduce monetary policy as a source of wholesale funding. Monetary policy will therefore affect the weighted average cost of funds for intermediaries.

The production function combines labour and capital in a typical Cobb-Douglas function. Since labour supply is fixed, we normalize it to 1. Output Y_t is produced according to the following technology:

$$Y_t = Z_t K_{t-1}^{\theta} L_t^{1-\theta} \tag{2}$$

where Z_t represents total factor productivity and θ the capital share of output. Given $L_t = 1$, in equilibrium firm maximization implies that wages $W_t = (1-\theta) Z_t K_{t-1}^{\theta-1}$. We will introduce some idiosyncratic risk to financial intermediation, so the return on a unit of capital will be intermediary specific $R_{it}^K = \theta Z_{it} K_{t-1}^{\theta-1} + (1-\delta)$ (more on this later).

The household program can be written as follows:

$$\max_{\{C_t, S_t^H, D_t^H\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^H) \qquad \text{s.t.} \tag{3}$$

$$C_t^H + D_t^H + S_t^H = R_t^D D_{t-1}^H + S_{t-1}^H + W_t - T_t \qquad \forall_t \tag{4}$$

where β is the subjective discount factor and $u(.)$ the period utility function. T_t are lump sum taxes and S_t^H are savings invested in the one-to-one storage technology. Note that the return on deposits is risk-free despite the possibility of intermediary default. The reason is that deposits are guaranteed by the government, which may need to

[14] Given households are risk-averse and intermediaries are risk neutral (and engage in risk-shifting), relaxing the assumption households cannot invest directly would make no difference to their portfolio in equilibrium unless all intermediaries are constrained. There are also little hedging properties in the asset, since the correlation of the shock to returns with wage income is positive. In the numerical exercises it is never the case that all intermediaries are constrained as some choose not to leverage, so to simplify notation and clarify the household problem, we assume directly that only intermediaries can invest in the risky capital stock.

raise taxes T_t in the event intermediaries cannot cover their liabilities[15]. Households understand that the higher the leverage of intermediaries, the more likely it is for them to be taxed in the future. However, they do not internalize this in their individual portfolio decisions since each household cannot by itself change aggregate deposits nor the expectation of future taxes. The return on storage is also risk-free, which implies that households will be indifferent between deposits and storage if and only if $R_t^D = 1$. Therefore, they will not save in the form of deposits if $R_t^D < 1$ and will not invest in storage if $R_t^D > 1$. In equilibrium, the deposit rate will be bounded from below by the unity return on storage, implying that $R_t^D \geq 1$. In the case $R_t^D = 1$, the deposit quantity will be determined by financial intermediary demand, with the remaining household savings being allocated to storage.

3.2 Financial intermediaries

The financial sector is composed of two-period financial intermediaries which fund themselves through inside equity and household deposits[16]. They use these funds to invest in the aggregate risky capital stock and/or in the riskless one-to-one storage technology. Intermediaries are risk neutral agents who maximize expected second period consumption subject to a Value-at-Risk (VaR) constraint. They also benefit from limited liability. To capture the diversity of risk attitudes among financial intermediaries, we assume that they are heterogeneous in α^i, the maximal probability their return on equity is negative according to their VaR constraint. α^i is exogenously given and is the key parameter in the VaR constraint. This probability varies across intermediaries and is continuously distributed according to the measure $G(\alpha^i)$ with $\alpha^i \in [\underline{\alpha}, \overline{\alpha}]$[17].

The balance sheet of intermediary i at the end of period t is as follows:

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[15]Before 2008, *stricto sensu* only deposit taking institutions were guaranteed. However, financing conditions of investment banks were very favourable and similar to the deposit taking institutions so that one interpretation of the data is that the market believed they were implicitly guaranteed. And in fact the market got largely proved right (except for Lehman) as Goldman Sachs, JP Morgan and others transformed their status into Bank holding companies to access the Fed lending facilities.

[16]We will extend the funding options to include wholesale funding, whose cost is influenced by monetary policy, in section 5. The economy in our benchmark case does not feature an interbank market or other funding possibilities. We relax this assumption and allow for interbank market in Appendix E. Qualitative results are unchanged.

[17]Assuming a continuous distribution is consistent with the smooth behaviour of leverage across quantiles shown in Figure 1. It also allows us to exploit the full panel of intermediaries balance sheets.

Assets	Liabilities
k_{it}	ω_{it}
s_{it}	d_{it}

where k_{it} are the shares of the aggregate capital stock held by intermediary i, s_{it} the amount of storage held, d_{it} the deposit amount contracted at interest rate r_t^D, and ω_{it} the inside equity. At the beginning of the next period, aggregate and idiosyncratic shocks are revealed and the net cash flow $\pi_{i,t+1}$ is:

$$\pi_{i,t+1} = R_{i,t+1}^K k_{it} + s_{it} - R_t^D d_{it} \tag{5}$$

Intermediary return on capital $R_{i,t+1}^K$ is risky and depends on the ex-post productivity of the capital held by the intermediary. It features an idiosyncratic and an aggregate productivity component. With probability ζ, the intermediary is hit by a negative idiosyncratic shock and its capital fails to produce anything, although it still recovers undepreciated capital at $t+1$. With probability $(1-\zeta)$ it is not hit by the negative idiosyncratic shock.[18] We can then describe idiosyncratic returns $R_{i,t+1}^K$ as follows:

$$R_{i,t+1}^K = \begin{cases} 1-\delta & \text{with probability} \quad \zeta \\ \theta \tilde{Z}_{t+1} K_t^{\theta-1} + (1-\delta) & \text{with probability } 1-\zeta \end{cases} \tag{6}$$

where $\tilde{Z}_t$ is the aggregate component and can be interpreted as the productivity of capital conditional on no idiosyncratic shock. ζ is a measure of idiosyncratic risk. The aggregate component follows a simple AR(1) process in logs

$$\log \tilde{Z}_{t+1} = (1-\rho^z)\mu_z + \rho^z \log \tilde{Z}_t + \varepsilon_{t+1}^z \tag{7}$$

$$\varepsilon_{t+1}^z \sim N(0, \sigma_z) \tag{8}$$

ε_t^z is the shock to the log of exogenous productivity (conditional on no idiosyncratic shock) with persistence ρ^z and standard deviation σ_z. μ_z is a scaling parameter such

[18] We can think of ζ as an operational risk shock. It is mainly introduced for computational purposes in order to ensure that the lowest (positive) probabilities of default of leveraged intermediaries are never numerically indistinguishable from zero. It also decouples the volatility of the return on assets from the volatility of the aggregate capital stock, which would otherwise be too low to generate at the same time realistic values for leverage and probabilities of default.

that $E(Z) = E(\tilde{Z}(1-\zeta)) = 1$. Let $F(\epsilon_t^z)$ be the cumulative distribution function (cdf) of $\exp(\epsilon_t^z)$, a notation which will be convenient later. Expected return on capital will be equal across intermediaries and we define $E\left[R_{t+1}^K\right] \equiv E\left[R_{i,t+1}^K\right]$. Differences in willingness to pay for shares of the capital stock will however arise in the presence of heterogeneous default risk and limited liability, generating an intermediary-specific option value of default.

3.2.1 Value-at-Risk constraint

Financial intermediaries are assumed to be constrained by a *VaR* condition. *VaR* are commonly used in risk management across a wide range of intermediaries (Stulz, 2016)[19]. This constraint imposes that intermediary i invests in such a way that the probability its return on equity is negative must be smaller than an exogenous intermediary-specific parameter α^i.[20] The *VaR* constraint for intermediary i can then be written as:

$$\Pr(\pi_{i,t+1} < \omega_{it}) \leq \alpha^i \tag{9}$$

The probability that the net cash flow is smaller than starting equity ω_{it} must be less or equal than α^i. This constraint follows the spirit of the Basel Agreements, which aim at limiting downside risk and preserving an equity cushion. Furthermore, Value-at-Risk techniques are used by banks and other financial intermediaries (for example asset managers) to manage risk internally. When binding, it also has the property of generating procyclical leverage, which can be observed in the data for some intermediaries as described in Geanakoplos (2011) and Adrian and Shin (2014) when equity is measured at book value. Using a panel of European and US commercial and investment banks Kalemli-Özcan, Sorensen and Yesiltas (2012) also provide evidence of procyclical leverage while emphasizing cross-sectional variations across types of intermediaries. Heterogeneity in the parameter of the *VaR* constraint can be rationalized in different ways. It could be understood as reflecting different risk management practices

[19]For a micro-foundation of VaR see Adrian and Shin (2014). For a macroeconomic model using VaR constraints, see Coimbra (2020).

[20]Alternatively we could posit that the threshold is at a calibrated non-zero return on equity. There is a mapping between the distribution $G(\alpha^i)$ and such a threshold, so for any value we could find a $\tilde{G}(\alpha^i)$ that would make the two specifications equivalent given expected returns. We decide to use the current one as it reduces the parameter space.

or differentiated implementation of regulatory requirements by different supervisors. For example, the Basel Committee undertook a review of the consistency of risk weights used when calculating how much capital a sample of banks put aside for precisely defined portfolios. When given a diversified test portfolio the banks surveyed produced a wide range of results in terms of modelled *VaR* and gave answers ranging from 13 million to 33 million euros in terms of capital requirement with a median of about 18 million (see Basel Committee on Banking Supervision (2013) p.52). Some of the differences are due to different models used, some to different discretionary requirements by supervisors and some to different risk appetites, as "Basel standards deliberately allow banks and supervisors some flexibility in measuring risks in order to accommodate for differences in risk appetite and local practices" (p.7). In the data, leverage is highly heterogeneous in the cross-section of financial intermediaries as can be seen in the descriptive statistics in Table A3 of appendix D.

3.2.2 Intermediary investment problem

We assume that the risk neutral intermediaries live for two periods, receive a constant endowment of equity $\omega_{it} = \omega$ in the first and consume their net worth in the second. This assumption of constant equity is a simplifying assumption but we find that book value equity is indeed very sticky in the data. We show in the left panel of Figure A3 the almost one-for-one correlation between changes in the size of debt and assets at book value, for a very broad sample of banks using Bankscope data. This panel also shows the stickiness of book value equity relative to assets and debt. Balance sheet expansions and contractions tend to be done through changes in debt and not through movements in equity. Krishnamurthy and Vissing-Jorgensen (2015) present remarkable evidence on the time series of bank long-term assets, short-term debt and equity as a percentage of GDP for the US. We replicate their findings in the right panel of Figure A3, highlighting the very strong correlation between long-term assets and short term debt (0.994) and a far smaller one between equity and assets (0.283). In addition, if we detrend the series, the correlations are, respectively, 0.972 and -0.02174 so still very high for assets and debt but virtually zero between equity and assets. Furthermore, the magnitudes of long term assets and short term debt are comparable throughout, highlighting the central role of leverage in funding investment in the economy. The macro-finance literature often focuses on the dynamics of net worth, assuming a representative agent (see e.g. Gertler and Kiyotaki (2015)), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2014), Jermann and Quadrini (2012)) and abstracting from the cross sectional differences in intermediaries. We take a complementary approach. To highlight the novel nature of our mechanism, we instead assume constant equity, thus abstracting from the net worth channel and putting a sharp focus on the effects generated by the heterogeneous dynamics of leverage in the cross-section.

When the net cash flow $\pi_{i,t}$ is positive, it is consumed by financial intermediary i and we denote its consumption by c_{it}.[21] When the net cash flow is negative, $c_{it} = 0$ and the intermediary defaults. Government steps in to repay depositors as it upholds deposit insurance. This is a pure transfer, funded by a lump sum tax on households. Hence, in our model, households are forward-looking and do intertemporal optimization while most of the action in the intermediation sector comes from heterogeneous leverage and risk taking in the cross-section. This modelling choice is made for simplicity and allows us to focus on the role of different leverage responses across financial intermediaries[22].

Each intermediary has to decide whether it participates or not in the market for risky assets or invests in the storage technology (*participating intermediary* versus *non-participating intermediary*) and, conditionally on participating, whether it uses deposits to lever up (*risky intermediary*) or just invests its own equity (*safe intermediary*). Note that this label of *risky* or *safe* is based on the possibility (or not) of defaulting on lenders, not in terms of the volatility of their return on assets or equity. These will only be risk free for *non-participating* ones, which invest only in storage. In Appendix E, we show that an alternative model where intermediaries can choose to lend to each other as an outside option has very similar implications.[23]

Intermediaries are assumed to be (constrained) risk-neutral price takers, operating in a competitive environment. Each maximizes consumption over the next period by picking k_{it} (investment in risky assets) and s_{it} (investment in the storage technology),

[21] When intermediary j is inactive, then $c_{jt} = \omega$ as the return of the storage technology is one.

[22] Other papers in the literature have used related assumptions, for example exogenous death of intermediaries in Gertler and Kiyotaki (2015).

[23] In Appendix E, we consider a standard centralized market for intermediary borrowing. For a model of financial stability with banking networks see Aldasoro, Gatti and Faia (2017).

under its VaR constraint, while taking interest rates on deposits R_t^D and asset return distributions $R_{t+1}^K(\varepsilon)$ as given. The program of each intermediary i is given by:

$$V_{it} = \max \ \mathbb{E}_t(c_{i,t+1}) \tag{10}$$

$$\text{s.t. } \Pr(\pi_{i,t+1} < \omega_{it}) \leq \alpha^i \tag{11}$$

$$k_{it} + s_{it} = \omega_{it} + d_{it} \tag{12}$$

$$c_{i,t+1} = \max\left(0, \pi_{i,t+1}\right) \tag{13}$$

$$\pi_{i,t+1} = R_{i,t+1}^K k_{it} + s_{it} - R_t^D d_{it}$$

where α^i is the VaR threshold (the maximum probability of not being able to repay stakeholders fully) and $\pi_{i,t+1}$ the net cash flow.

Intermediaries can choose not to lever up ($d_{it} = 0$) or even stay out of capital markets and not participate ($k_{it} = 0$). In this case, they have the outside option of investing all their equity in the storage technology and collect it at the beginning of the next period. The value function of a non-participating intermediary investing in the outside option is:

$$V_{it}^O = V^O = \omega \tag{14}$$

3.2.3 Limited liability

The presence of limited liability truncates the profit function at zero, generating an option value of default that intermediaries can exploit. For a given expected value of returns, a higher variance increases the option value of default as intermediaries benefit from the upside but do not suffer from the downside. For a given choice of k_{it} and d_{it} we have that:

$$\mathbb{E}_t\left[\max(0, \pi_{i,t+1})\right] \geq \mathbb{E}_t\left[\pi_{i,t+1}\right] \tag{15}$$

with the inequality being strict whenever the probability of default is strictly positive. Deposit insurance transfers t_t^i happen when the net cash flow is negative and are given by:

$$t^i_{t+1} = \max\left(0, -\pi_{t+1}\right) \tag{16}$$

The max operator selects the appropriate case depending on whether intermediary i can repay its liabilities or not. If it can, then deposits repayments are lower than return on assets and deposit insurance transfers are zero. Total intermediary consumption C^I_t and aggregate transfers/taxes T_t are given by integrating over the mass of intermediaries:

$$C^I_t = \int c_{it}\ dG(\alpha^i) \tag{17}$$

$$T_t = \int t^i_t\ dG(\alpha^i) \tag{18}$$

For now we assume default is costless in the sense that there is no deadweight loss when the government is required to pay deposit insurance. In section 6, we will drop the assumption of costless default by having a more general setup that allows for a lower return on assets held by defaulting intermediaries.

3.3 Investment strategies and financial market equilibrium

Financial intermediaries are price takers, therefore the decision of each one depends only on the expected return on assets (taking into account limited liability) and the cost of liabilities. Since the mass of each intermediary is zero, individual balance sheet size does not affect returns on the aggregate capital stock. Intermediary i will be a *participating intermediary* in the market for risky assets whenever $V_{it} \geq V^O$. This condition determines entry and exit into the market for risky capital endogenously.

There is however another important endogenous decision. Intermediaries which participate in the market for risky assets have to choose whether to lever up and, if they do, by how much. We will refer to the decision to lever up or not, i.e. to enter the market for deposits as the *extensive margin*. We will refer to the decision regarding how much to lever up as the *intensive margin*. Financial intermediaries which lever up are *risky* intermediaries. Financial intermediaries which participate in the market for risky capital but do not lever up are *safe* intermediaries.

Proposition 3.1 *When $\mathbb{E}[R^K_{t+1}] \geq 1$, participating intermediary i will either lever up to its VaR constraint or not raise deposits at all.*

Proof: See Appendix B.

Proposition 3.1 states that if the return to risky capital is higher in expectation than the return on the storage technology then whenever an intermediary decides to lever up, it will do so up to its *VaR* constraint and will not invest in storage. Hence all *risky* intermediaries will be operating at their constraint.

When expected return on risky capital is smaller than return on storage: $\mathbb{E}[R^K_{t+1}] < 1$, it might still be the case that capital is preferred to storage in equilibrium by some intermediaries due to limited liability. We would then have equilibria in which some intermediaries invest in storage and possibly some of the most risk-taking ones leverage up a lot taking advantage of the option value of default. In what follows we focus on cases where $\mathbb{E}[R^K_{t+1}] \geq 1$ which is always the case in our simulations.

3.3.1 Intensive margin and endogenous leverage

Let $Z^e_{t+1} \equiv \mathbb{E}_t(\tilde{Z}_{t+1})$, an expectation known at t. For a participating intermediary i deciding to lever up, the *VaR* condition will bind (see Proposition 3.1):

$$\Pr\left[\pi^i_{t+1} \leq \omega\right] = \alpha^i \tag{19}$$

Hence, after some straightforward algebra, we obtain the following:

$$\zeta + (1-\zeta)\Pr\left[e^{\varepsilon^z_{t+1}} \leq \frac{r^D_t + \delta - \frac{\omega}{k_{it}} r^D_t}{\theta Z^e_{t+1} K^{\theta-1}_t}\right] = \alpha^i \tag{20}$$

The leverage λ_{it} of an active intermediary is given by:

$$\lambda_{it} \equiv \frac{k_{it}}{\omega} = \frac{r^D_t}{r^D_t - \theta Z^e_{t+1} K^{\theta-1}_t F^{-1}\left(\frac{\alpha^i-\zeta}{1-\zeta}\right) + \delta} \tag{21}$$

where we defined leverage as assets over equity and F^{-1} as the inverse cdf of the technology shock $e^{\varepsilon^z_{t+1}}$ evaluated at probability $\frac{\alpha^i-\zeta}{1-\zeta}$. Note that intermediaries with

$\alpha^i < \zeta$ will never participate.

Let $r_t^{\alpha^i} \equiv \theta Z_{t+1}^e K_t^{\theta-1} F^{-1}\left(\frac{\alpha^i-\zeta}{1-\zeta}\right) - \delta$ be the ex-post return on capital for which the return on equity of *risky* intermediary α^i is zero. The expression above can then be simply written as:

$$\lambda_{it} = \frac{r_t^D}{r_t^D - r_t^{\alpha^i}} \tag{22}$$

This expression for leverage is only true when the constraint is binding for *risky* intermediary α^i. In equilibrium, decreasing marginal returns to K ensure that the denominator is always positive. Otherwise the constraint would not be binding and *risky* intermediaries would increase K, which in turn would reduce $r_t^{\alpha^i}$.

Proposition 3.2 *For a participating intermediary i, the leverage λ_{it} has the following properties: it is increasing in α^i, decreasing in the cost of funds r_t^D and increasing in expected marginal productivity of capital $\theta Z_{t+1}^e K_t^{\theta-1}$. Furthermore, $\frac{\partial^2 \lambda_{it}}{\partial (r_t^D)^2} > 0$ and $\frac{\partial^2 \lambda_{it}}{\partial r_t^D \partial \alpha^i} < 0$.*

Proof: Immediate from Equation (21) and given the monotonicity of the cdf and the shape of $F^{-1}()$.

Proposition 3.2 implies that, from the perspective of a participating individual intermediary (i.e. absent general equilibrium effects on K_t), leverage will be decreasing in the cost of funds r_t^D. For a given balance sheet size, decreasing the cost of liabilities increases expected *net* cash flows and thus decreases the probability of distress. From 3.1, intermediaries would then choose to increase leverage until their probability of distress hits the VaR constraint. Furthermore, when interest rates are low the probability of default is lower *ceteris paribus*. In that region, the pdf is flatter therefore increases in leverage translate into small increases in probability of distress. This means that intermediaries can increase leverage by sizable amounts until they hit the *VaR* constraint. So the lower r_t^D, the stronger the intensive margin effect. Similarly for high α (looser constraints) leverage can be increased a lot before the constraint is hit. Therefore the leverage of the most risk-taking intermediaries will react more to interest rate changes. This heterogeneity of the *intensive margin* to changes in the cost of funds means that as interest rates fall, the more skewed will be the distribution of leverage in the cross-section. This generates a composition effect, where the proportion of assets being held by the more risk-taking intermediary rises. Since the intensive margin effect is larger the lower are interest rates to begin with, it follows that this composition effect is particularly strong at low levels of interest rates.

Generally, intermediary leverage will also be decreasing in the volatility of the productivity shocks σ_z. This will be true whenever $F^{-1}\left(\frac{\alpha^i-\zeta}{1-\zeta}\right)$ is increasing in σ_z, implying realistically that the probability of a negative return on equity is (ceteris paribus) increasing in the volatility of returns.

3.3.2 Extensive margin and endogenous leverage

We now focus on the *extensive margin*, that is to say whether intermediaries who participate in risky capital markets choose to lever up using deposits or not.[24]

Let V^L denote the value function of *risky* intermediaries who decide to lever up using deposits and V^N the value function of the *safe* ones who only invest at most their equity in the risky capital stock.

$$V_{it}^L = \mathbb{E}_t\left[\max\left(0, R_{i,t+1}^K k_{it} - R_t^D d_{it}\right)\right] \tag{23}$$

$$V_{it}^N = \mathbb{E}_t[R_{i,t+1}^K] k_{it}^N + \omega - k_{it}^N \tag{24}$$

with $k_{it}^N \in \{0, \omega\}$ and the max operator in V^L being the effect of limited liability. Since there is no risk of defaulting on deposits if you have none, there is no option value of default for non-levered intermediaries. This N group could in principle also include intermediaries who invest only a fraction of their equity in the capital stock. Given our choice of *VaR* constraint, *safe* intermediaries will either invest ω in the capital stock or not at all.[25]

We can then use the condition $V_{it}^L = V_{it}^N$ to find the cut-off value $\alpha_t^L = \alpha_t^j$ for

[24] Intermediaries can also decide not to invest in risky capital markets and instead to use the storage technology. If they do so, then their value function is $V^O = \omega$ given the unit return to storage and linear utility.

[25] Note that the *VaR* condition of a *safe* intermediary can be written as $\Pr\left(e^{\varepsilon_{t+1}} < \frac{\delta K^{1-\alpha}}{\theta Z_{t+1}^e}\right) \leq \frac{\alpha^i-\zeta}{1-\zeta}$. Since this is not a function of k_{it}, the inequality will either be true and the intermediary will invest up to ω, or it won't and he cannot invest any amount in the capital stock without violating it. Note also that the inequality is always false for $\alpha^i < \zeta$.

which intermediary j is indifferent between leveraging up or not. Above α_t^L (looser *VaR* constraints), all intermediaries will be levered up to their respective constraints and do not invest in storage as shown in Proposition 3.1. For any levered intermediary i, we have:

$$\mathbb{E}\left[\max\left(0, k_{it}R_{i,t+1}^K - R_t^D d_{it}\right)\right] \geq \omega\ \mathbb{E}_t\left[R_{t+1}^K\right] \tag{25}$$

where the left hand side is the expected payoff on the assets of intermediary i and the right hand side is the expected payoff when it invests only its equity ω in capital markets. Using the balance sheet equation $k_{it} = d_{it} + \omega$, we can substitute for deposits, which leads to the following condition:

$$\mathbb{E}_t\left[\max\left(0, k_{it}\left(R_{i,t+1}^K - R_t^D\right) + R_t^D\omega\right)\right] \geq \omega\ \mathbb{E}_t\left[R_{t+1}^K\right] \tag{26}$$

For the marginal intermediary j, equation (26) holds with equality:

$$\mathbb{E}_t\left[\max\left(0, k_{jt}\left(R_{j,t+1}^K - R_t^D\right) + R_t^D\omega\right)\right] = \omega\ \mathbb{E}_t\left[R_{t+1}^K\right] \tag{27}$$

Since all *risky* intermediaries will be at the constraint, we can combine equation (27) with equation (21) evaluated at the marginal intermediary (whose *VaR* parameter is α_t^L). Moreover, $\mathbb{E}_t\left[R_{t+1}^K\right]$ is a function of Z_{t+1}^e and K_t but is independent of i. Therefore equation (27) and equation (21) jointly define an implicit function of the threshold *VaR* parameter α_t^L $(= \alpha^j)$ with variables (r_t^D, Z_{t+1}^e, K_t).

Hence we have the following result:

Proposition 3.3 *There exists a cut-off value α_t^L in the distribution of VaR parameters such that all intermediaries with VaR constraints looser than the cut-off will borrow to leverage up to their constraint. All intermediaries with VaR constraints tighter than the cut-off will choose to not leverage. Equations (27) and (21) define an implicit function of the threshold $\alpha_t^L = A(r_t^D, Z_{t+1}^e, K_t)$.*

3.3.3 Financial market equilibrium and deposit demand curve

To close the financial market equilibrium, we need to use the market clearing condition. The aggregate capital stock of the economy is equal to the total investment in risky projects by all intermediaries.

$$K_t = \int_{\underline{\alpha}}^{\overline{\alpha}} k_{it} \; dG(\alpha^i) \tag{28}$$

This integral can be divided into capital held by *risky* levered intermediaries (above α_t^L) and capital held by *safe* intermediaries who do not lever up but invest all their equity in the capital stock (between α_t^N and α_t^L). Below α_t^N all intermediaries invest all their equity in storage.

For *safe* intermediaries who invest all their equity in capital shares, the VaR constraint is given by $\zeta + (1-\zeta)F\left(\frac{\delta K_t^{1-\theta}}{\theta Z_{t+1}^e}\right) \leq \alpha^i$. We can pin down α_t^N by looking at the marginal *safe* intermediary for whom the constraint binds exactly.

$$\alpha_t^N = \zeta + (1-\zeta)F\left(\frac{\delta K_t^{1-\theta}}{\theta Z_{t+1}^e}\right) \tag{29}$$

In equilibrium, the market clearing condition for K can then be written as:

$$K_t = \int_{\alpha_t^L}^{\overline{\alpha}} k_{it} \; dG(\alpha^i) + \left[G(\alpha_t^L) - G(\alpha_t^N)\right] \omega \tag{30}$$

Where k_{it} is given by the asset purchases of *risky* intermediaries described in equation (21). Along with the expression for α_t^N in equation (29), the market clearing equation (30) defines an implicit function of $(\alpha_t^L, r_t^D, Z_{t+1}^e, K_t)$. Since Z_{t+1}^e is determined at t by state variables and intermediaries are price takers, the financial market clearing function together with the implicit function $\alpha_t^L = A(r_t^D, Z_{t+1}^e, K_t)$ pin down the aggregate capital stock K_t and the marginal levered intermediary α_t^L, for a given deposit rate r_t^D and expected productivity Z_{t+1}^e.

Together they determine the *aggregate demand curve for deposits* as a function of deposit rates and expected productivity. By pinning down (α_L, K), they also determine the entire distribution of leverage in the financial sector for a given (r_t^D, Z_{t+1}^e). In general equilibrium, described in section 5, the deposit rate r_t^D will be determined in conjunction with the *aggregate deposit supply curve* coming from the recursive household problem.

3.4 Measuring Financial Stability

The model establishes an important relation between funding costs and the cross-sectional distribution of risk taking by financial intermediaries. Financial stability is a multidimensional object depending on time-varying distributions of leverage and risk taking which are functions of present and future states. For expositional purposes, we summarize this object into a few simple but relevant measures of financial instability in order to track its evolution.

Our baseline measure M^1 is *the probability that in the next period all leveraged intermediaries will be in distress*, defined as the inability to repay in full their stakeholders (deposits and equity). This has a very direct link with the *VaR* constraint, as for each levered intermediary the probability of distress will be simply the parameter α^i. Given aggregate shocks by definition affect all intermediaries, then $M_t^1 = \alpha_t^L$. If the least risk-taking leveraged intermediary is in distress, so must all the intermediaries with higher leverage.[26] In the model, a rise in α_t^L (meaning that the marginal entrant has a looser *VaR* constraint) is then a fall in financial stability according to M^1. The baseline measure has the advantage of not only describing the risk of the whole sector but also of tracking the marginal investor in financial markets, an important concept in leverage cycles, as highlighted by Geanakoplos (2011).

The model features significant risk-shifting behavior, as levered financial intermediaries take advantage of limited liability and the option value of default. Moreover, the riskier the intermediary, the larger will be their option value of default. To have a sense of aggregate distortions to investment caused by risk-shifting, we calculate a *Weighted Option Value of Default* by weighing each intermediary's option value of default by their total assets. This measure M^2 can therefore be interpreted as the average option value of default per unit of capital in the economy. In the following sections we will

[26]More precisely, in the presence of idiosyncratic shocks M^1 would be an affine transformation of α_t^L, with $M_t^1 = \zeta + (1-\zeta)\alpha_t^L$. Given this transformation is time-invariant, for simplicity we set $M^1 = \alpha_t^L$ even in the case with idiosyncratic shocks.

use measures M^1 and M^2 to track the dynamics of financial stability in response to monetary and productivity shocks.[27]

4 Financial sector equilibrium

To provide intuition on how the financial sector works in the model, we first show a set of partial equilibrium results taking as given the deposit rate, before moving on to general equilibrium in section 5 where the household problem will close the model. From now on we study the properties of the model using numerical simulations.[28]

We begin by analysing the distribution of intermediary leverage conditional on the deposit rates r_t^D and on expected productivity Z_{t+1}^e. In Figure 5, we show an example of the cross-sectional distribution of leverage for three different values of the deposit rate. The calibration of the model is discussed in more detail in section 5.

In the three cases, the area below each line[29] is proportional to the aggregate capital stock $K_t = \int k_{it}\ dG(\alpha^i)$. The vertical line showing a drop in leverage marks the cut-off and identifies the marginal levered intermediary α_t^L. To the left of the cut-off α_t^L, intermediaries are not levered, which corresponds to the more conservative VaR constraints. They are the *safe* intermediaries. To the right of the cut-off, leverage and balance sheet size k_{it} increase with α_t^i. That is, the more risk-taking is the intermediary, the larger will be its balance sheet for a given r_t^D and Z_{t+1}^e. Those are *risky* intermediaries.

The graph illustrates how the *intensive* and *extensive* margins affect leverage and the aggregate capital stock as the deposit interest rate changes. For the three cases, as deposit rates fall, the *intensive* margin for the most risky intermediaries is increasing. That is, for the riskiest intermediaries, the balance sheet grows when the cost of funds falls. For a given balance sheet size, a lower rate would reduce the probability of default as it reduces the amount that needs to be repaid next period. This relaxes the *VaR*

[27] Since we can describe the whole cross-sectional distribution of leverage and intermediary risk we can also use a range of potential alternative measures. We highlight this point by providing 3 other measures of systemic risk in Appendix C.

[28] We performed many different calibrations but only report a few. Results (available upon request) are qualitatively robust across simulations.

[29] Assuming a uniform distribution for $G(\alpha^i)$ as in the baseline calibration. The details of the numerical method to solve the model are given in Appendix A.

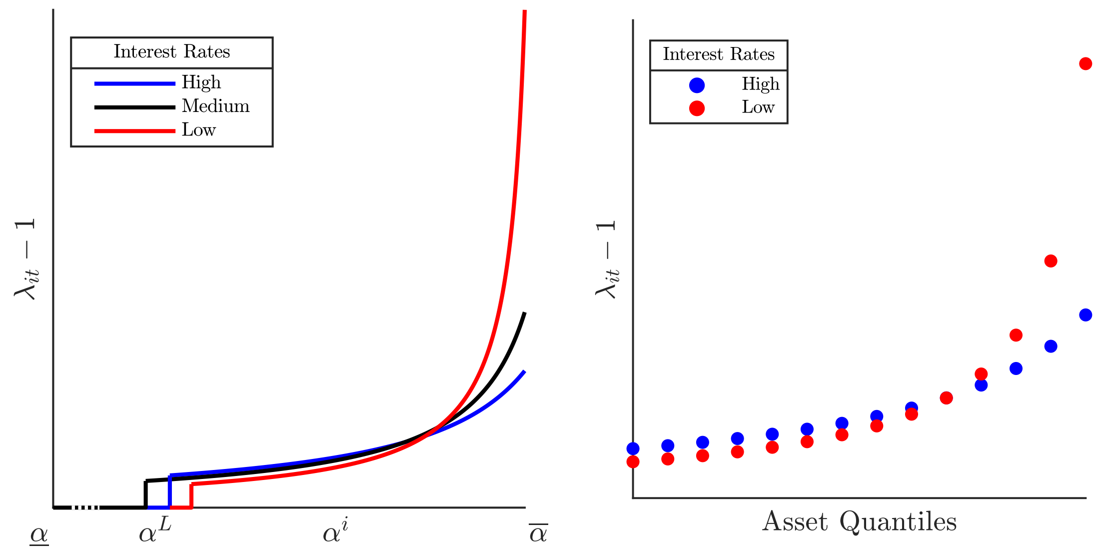

Figure 5: *Cross-sectional distribution of leverage* λ_{it} *as a function of the VaR parameter* α^i *(left panel) and quantiles of asset size among leveraged institutions (right panel).* $\lambda_{it} - 1$ *is used so that unlevered participants have a value of 0.*

constraint, so intermediaries at the top of the distribution expand their balance sheet up to the new limit and grow in size. Perhaps less intuitively, the effect for intermediaries in the middle of the distribution and on the *extensive* margin is ambiguous. One would expect that a fall in interest rate would lead to higher leverage by all intermediaries and entry of more risk averse intermediaries. Entry does occur when one goes from a high level of interest rate to a medium level of interest rate (the cut-off moves to the left). But this is no longer the case when one moves from a medium level of interest rate to a low level of interest rate: the cut-off moves to the right. Depending on the level of interest rates, a fall in interest rates can lead to more or fewer intermediaries choosing to lever up. The intensive margin effect at the middle and tail of the distribution is also not positive, in fact it decreases for many intermediaries. This is due to the fall in expected returns as K increases, driven by the large balance sheet expansion of the riskier intermediaries. We explain below this strong non-linearity of the effect of interest rates on financial stability and the leverage of intermediaries in the middle of the distribution.

On the right panel, we show the model implied leverage per asset quantile of levered institutions for high and low rates. Leverage among lower quantiles does not vary much across the two cases, while those at the top exhibit significantly larger leverage whenever interest rates are low. Those patterns are consistent with the stylized facts described in Section 2.

4.1 Non-linear trade-off between economic activity and financial stability

Following a fall in deposit rates, the riskier intermediaries expand their asset holdings raising the aggregate capital stock. This lowers the return on risky asset holdings due to decreasing returns to (aggregate) capital. As seen in the graph above, this *pecuniary externality* gives very interesting asymmetries depending on the *level* of the interest rate.

When the interest rate level is high, the lower cost of liabilities reduces the probability of default for a given balance sheet size. Hence all intermediaries with a risky business model are able to lever more (*intensive* margin). In this case, there are also positive returns for the (previously) marginal intermediary due to the now lower cost of leverage. More intermediaries can lever up and enter the market for deposits (*extensive* margin), reducing the cut-off α^L. The financial system then becomes less risky since newly entered intermediaries have a stricter *VaR* constraint. According to measure $M^1 = \alpha^L$ there is no trade-off in this case between using lower interest rates to stimulate investment and financial stability.

When the interest rate level is low, the *intensive* margin effect of a decrease in the interest rate is strong (see Proposition 3.2), leverage and investment are high and the curvature of the production function leads to a decrease in expected asset returns which is large enough to price out of the market the most risk averse of the previously levered intermediaries. The sign of the effect on α^L depends on whether the fall in asset returns is stronger than the fall in the cost of liabilities. In the case of initially low interest rates, a further fall (in those rates) leads to fewer intermediaries choosing to lever up. The intermediaries who leverage up are larger and more risk taking on average. There is therefore a clear trade-off between an expansionary monetary policy (that lowers funding costs) and financial stability.

In order to gain some intuition, we differentiate the financial market clearing condition. For simplicity, assume $\alpha^N \sim 0$ so we can rewrite (30) as:

$$K = \omega + \omega \int_{\alpha^L}^{\overline{\alpha}} (\lambda^\alpha - 1)\; dG(\alpha) \tag{31}$$

Normalizing $\omega = 1$ and taking derivatives with respect to r we have:

$$\frac{\partial \alpha^L}{\partial r}(\lambda^{\alpha^L} - 1) = \int_{\alpha^L}^{\overline{\alpha}} \frac{\partial \lambda^\alpha}{\partial r} dG(\alpha) - \frac{\partial K}{\partial r} \tag{32}$$

where we can interpret $\frac{\partial \alpha^L}{\partial r}$ as the effect on the extensive margin and $\frac{\partial \lambda^\alpha}{\partial r} < 0$ as the intensive margin effect, summed over the set of participating intermediaries. $\frac{\partial K}{\partial r} < 0$ is the (endogenous) elasticity of capital to interest rates. If the intensive margin effect is weak and/or the elasticity of capital to interest rate is high then more intermediaries decide to lever up $\frac{\partial \alpha^L}{\partial r} > 0$. If it is strong and/or the elasticity of capital is low then fewer intermediaries lever up. According to proposition 3.2, the intensive margin effect is larger the lower interest rates are. Hence the lower the funding costs, the more likely the intensive margin dominates the right hand side of (32) and $\frac{\partial \alpha^L}{\partial r} < 0$: we have a rise in α^L when the interest rate goes down (from a low level), which means more financial instability.

The sign of the extensive margin effect $\frac{\partial \alpha^L}{\partial r}$ depends also on the elasticity of the capital stock with respect to the interest rate, which governs the strength of the pecuniary externality. In the extreme case where aggregate capital is fixed ($\frac{\partial K}{\partial r} = 0$), only returns adjust to clear the market[30] and we can see from Equation (32) that the cutoff must rise when rates fall. Since a fall in the cost of funding allows more leverage from the more risk-taking intermediaries, then it must be that the (previously) marginal intermediary no longer holds capital and returns will fall enough to price it out. In this case, there will always be a trade-off between increased investment and financial stability. Housing markets where the supply of capital is relatively inelastic

[30] In this case, the price of capital will adjust as it is no longer pinned down by the investment technology. For recent macroeconomic models in which extensive and intensive margin have interesting interactions (albeit in very different contexts) see Martin and Ventura (2015) and Bergin and Corsetti (2015).

are a possible example of such a case. In the polar extreme case where aggregate capital is infinitely elastic[31], a decrease in the cost of funding can only lead to entry as the (previously) marginal intermediary will now make positive profits. The cut-off falls and there is no trade-off. In intermediate cases, the strength of the intensive margin effect will be the main determining factor. The stronger is this effect (i.e. the more leverage increases following a fall in interest rates or the more *interest-elastic* the intermediaries are), the more likely a trade-off will be present. As stated in Proposition 3.2, leverage is more responsive when the level of interest rates is low and for intermediaries with looser *VaR* constraints. Hence, as shown in Figure 5, when interest rates fall from high to medium to low, balance sheets become more heterogeneous in size and the difference between the most leveraged and the least leveraged intermediary rises. We highlight the following properties of our model:

1) Leverage skewness, aggregate investment and volatility paradox

In Figure 6, the left panel plots the cut-off α_t^L as a function of deposit rates r_t^D for three different productivity levels, while the middle panel does the same for the aggregate capital stock K_t. K_t is monotonically decreasing with r_t^D. As expected, the lower is the interest rate, the higher will be aggregate investment and we have a standard deposit demand curve. However, the change in *financial structure* underlying the smooth response in the capital stock is non-monotonic. As we can see from the left panel, the cut-off α_t^L first decreases when we go from high interest rates to lower ones and then goes up sharply as we approach zero. There is a change in the composition of intermediaries. Less risk-taking intermediaries reduce their exposure and decrease asset holdings as they are priced out by more risk-taking institutions due to decreasing returns to capital. The latter use low interest rates to increase their leverage significantly.

The lower is the interest rate, the more heterogeneous is leverage across intermediaries. Since the intensive margin of high α^i intermediaries responds more than for low α^i ones, when interest rates are low there is an increased concentration of assets in the most risk-taking intermediaries. In addition, a fall in the extensive margin is more likely at low rates, which amplifies this effect. In the right panel of Figure 6 we show the cross-sectional skewness of leverage is a decreasing function of the interest rate. The concentration of assets in riskier intermediaries generates more risk-shifting in aggregate.

[31] Return distributions $R_{t+1}^K(\varepsilon)$ are therefore independent of the quantity of capital.

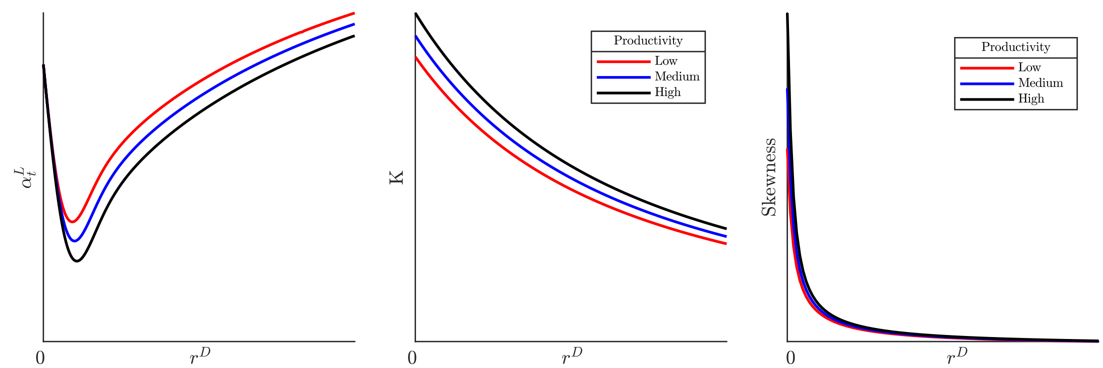

Figure 6: *Cut-off level* α_t^L *and aggregate capital stock as a function of deposit rates* r_t^D

Hence, similar aggregate investment outcomes can be supported by different underlying financial structures with very different implications for financial stability. Similarly, as shown in Figure A13 of Appendix F, a lower level of volatility of the productivity shocks increases concentration in the banking sector for a given level of funding costs as the more risk taking financial intermediaries can capture a higher market share. Therefore, low fundamental volatility periods should be associated *ceteris paribus* with high concentration in the banking sector and more systemic fragility, accounting for the *volatility paradox*[32] .

2) Trade-off between financial stability and economic activity

When interest rates are high, a fall in interest rates leads to entry by less risk-taking intermediaries (a fall in the cut-off α_t^L) into levered markets. But when interest rates are low, a fall in interest rates leads to a rise in the cut-off α_t^L, which means the least risk-taking intermediaries reduce their exposure to the risky asset through deleveraging, while the more risk-taking intermediaries increase their balance sheet size and leverage. We illustrate this point in our partial equilibrium setting by doing a 100 basis points monetary expansion for different target rates. As we will see in section 5, these results carry over to the general equilibrium setting. For this experiment, we assume a very

[32]The period 2003-2007 for example was a period of low fundamental volatility but where systemic risk was high, leading up to the global financial crisis of 2008. We note that is about comparative statics as we compare an economies with low and high volatility of fundamentals

simple monetary policy rule:

$$R_t = R_{t-1}^{\nu} \bar{R}^{1-\nu} \varepsilon_t^R \tag{33}$$

where $R_t = 1 + r_t^D$ is the return on deposit or the cost of leverage for intermediaries. ε_t^R is a monetary policy shock and ν is the persistence of the shock, calibrated[33] to 0.24. $\bar{R}$ is the long-run level of interest rates therefore each of the lines above is calibrated to a different $\bar{R}$. For simplicity, we assume that the monetary authority can directly affect the deposit rate. We relax this assumption in section 5 and show how it can be mapped into this exercise.

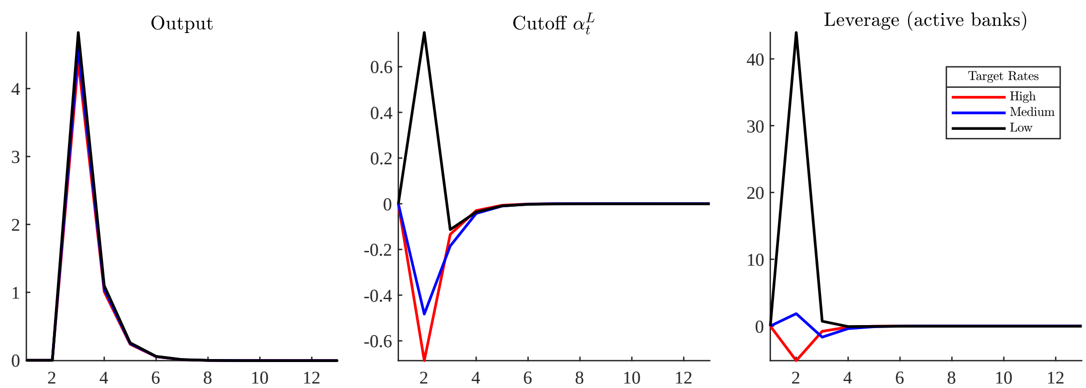

Figure 7: *Partial equilibrium IRF to a 100 basis points fall in deposit rates. Scale in percentage point deviations from the baseline*

Results can be seen in Figure 7, plotted as percentage changes from their respective values at target rates $\bar{R}$.[34] The time period corresponds to one year and the state of the economy when the shock hits is the one corresponding to the target rates. In the left graph we see that the rise in output is relatively insensitive to the level of the target interest rate. The behaviour of the cut-off α_t^L is, however, very differentiated. When the target rates are high, there is a negative effect of a monetary expansion on the cut-off.

[33] Annualized value as estimated by Curdia et al. (2015).

[34] Note that there is no true dynamics in the partial equilibrium model, which can be seen as a sequence of static problems. The general equilibrium model of section 5 will feature a fully dynamic household problem which affects the banking problem, since the household inter-temporal maximization will determine the deposit supply curve and the equilibrium level of deposit rates.

That means that less risk-taking intermediaries enter risky markets and the average probability of intermediary default falls. In this case, there is no trade-off between financial stability and monetary expansion. This is definitely not the case when target interest rates are low. In that case, average leverage of active banks increases massively by 43% and the cut-off also rises. The large increase in leverage by very risk-taking intermediaries then prices out the less risk-taking ones at the margin, raising the average probability of default among levered intermediaries. This large effect on leverage is a combination of both the intensive margin effect, and a composition effect due to exit of the most risk averse intermediaries. For intermediate levels, we see that this effect is muted, with leverage increasing only slightly and financial stability improving (cutoff going down). Hence, there is a trade-off between financial stability and monetary policy when interest rates are low, but not when they are high. The level of the interest rate matters since it affects the sensitivity of the intensive margin to changes in the cost of funds. The fact that risk-taking intermediaries are able to lever more when the cost of funds is low increases the capital stock and can price out of the market less risk-taking intermediaries. The financial sector becomes less stable, as risky assets concentrate in very large, more risk-taking financial institutions. There is also potentially a large mispricing of risk since the riskier intermediaries are those who engage the most in risk-shifting (measured in the aggregate by our measure of systemic risk M^2).[35]

We note that all the effects described above regarding the dispersion and the cyclicality of leverage, financial stability and aggregate risk-shifting can occur even in the absence of monetary policy shocks. Similar effects could be due to the deregulation of the financial sector (e.g. by increasing the looseness of the *VaR* constraints) or to low volatility environments. The cyclicality of the savings behaviour or of capital flows and their effect on equilibrium deposit rates will also lead to cyclical movements in leverage and investment. To understand this more fully, we now close the general equilibrium model by adding the intertemporally optimizing household sector to determine the deposit rate endogenously.

[35]Other measures of financial stability presented in Appendix C also highlight the presence of an important trade-off which occurs only at low levels of the interest rate.

5 General Equilibrium

In this section, we solve the model in general equilibrium by joining the household and intermediary problems. We show that the financial sector equilibrium can be easily integrated in a standard dynamic stochastic general equilibrium framework, with monetary policy and productivity shocks. We introduce costly default in section 6.

5.1 Monetary policy as a change in the cost of external funds

We allow intermediaries to fund themselves through wholesale funding l_{it}. We assume that the monetary authority can control the rate of wholesale funding relative to deposits, by providing funds at a spread γ_t from deposits.[36] Wholesale funding is remunerated at rate $R_t^L = 1 + r_t^L$ and we denote the deposit rate r_t^D as before. We assume that:

$$R_t^L = R_t^D(1 - \gamma_t) \tag{34}$$

Monetary policy is exogenous, akin to a funding subsidy γ_t which follows a simple AR(1) process in logs.

$$\log \gamma_t = (1 - \rho_\gamma)\mu_\gamma + \rho_\gamma \log \gamma_{t-1} + \varepsilon_t^\gamma \tag{35}$$

$$\varepsilon_t^\gamma \sim N(0, \sigma_\gamma) \tag{36}$$

where μ_γ is the central bank target subsidy, ρ_γ the subsidy's persistence and ε^γ are monetary policy shocks with σ_γ standard deviation. If the central bank were to provide unlimited funds to intermediaries at this rate, they would leverage using only wholesale funding. Wholesale funding is given in a fixed proportion χ of other liabilities, which in

[36]The monetary authority is assumed to be a deep-pocketed institution which can always fund wholesale funding. Like deposits, wholesale funds are always repaid (by bailout if necessary). To avoid dealing with the monetary authority's internal asset management, we assume that the cost of fund is a deadweight loss (or gain). Other transmission mechanisms of monetary policy, for example via price stickiness, are left for future research.

this case are simply deposits. Total wholesale funding for intermediary i is given by:

$$l_{it} = \chi d_{it} \tag{37}$$

The balance sheet of an intermediary i is then:

Assets	Liabilities
k_{it}	ω
s_{it}	d_{it}
	l_{it}

We can then define R_t^F as the total cost of a unit of funding and f_{it} as total external funds of bank i.

$$R_t^F = \frac{1+\chi(1-\gamma_t)}{1+\chi} R_t^D \tag{38}$$

$$f_{it} = (1+\chi) d_{it} \tag{39}$$

The balance sheet can be rewritten as follows:

Assets	Liabilities
k_{it}	ω
s_{it}	f_{it}

With external funds being remunerated at rate R_t^F. We obtain the same banking problem as before, replacing deposits by total funds f_{it} and the deposit rate by the unit cost of funds R_t^F. We can solve as before by mapping f_{it} and R_t^F easily into deposits d_{it} and their rate R_t^D. By moving γ_t the central bank will be able to change R_t^F as long as changes in equilibrium R_t^D do not offset perfectly the changes in the spread on the total cost of funding.

5.2 Solving the dynamic model

The financial sector equilibrium determines investment given funding costs R_t^F and expected productivity Z_{t+1}^e. We can then solve for the aggregate capital stock K and cut-off α_t^L as a function of R_t^F and expected productivity Z_{t+1}^e.

$$K = K^*(R^F, Z^e) \tag{40}$$

$$\alpha^L = \alpha^{L,*}(R^F, Z^e) \tag{41}$$

By integrating balance sheet equations, we obtain an expression for total funds F_t and deposit supply D_t:

$$F_t = \int_{\alpha_t^L}^{\overline{\alpha}} (k_{it} - \omega)\ dG(\alpha^i) \tag{42}$$

$$D_t = \int_{\alpha_t^L}^{\overline{\alpha}} d_{it}\ dG(\alpha^i) = \frac{F_t}{1+\chi} \tag{43}$$

where $F_t = \int f_{it}\ dG(\alpha^i)$ are total liabilities held by *leveraged* intermediaries and D_t is the aggregate deposit demand. Market clearing in the deposit market requires supply and demand to be equal.

$$D_t^H = D_t \tag{44}$$

Goods market clearing requires that output is used in consumption of intermediaries and households, investment and the accumulation of storage. The investment good is the consumption good and there are no capital or investment adjustment costs[37]. Aggregate investment I_t is given by the law of motion of the capital stock $K_t = (1-\delta)K_{t-1} + I_t$. The resource constraint of the economy is as follows:

$$S_{t-1}^H + S_{t-1}^I + Y_t = C_t^H + C_t^I + S_t^H + S_t^I + I_t + T_t^L \tag{45}$$

where $C_t^I = \int c_{it}\ dG(\alpha^i)$ and $T_t^L = \int l_{it}\ dG(\alpha^i) - R_{t-1}^L \int l_{i,t-1}\ dG(\alpha^i)$ is the net wholesale funding. S_t^H are the holdings of storage held by households and $S_t^I = \int s_{it}\ dG(\alpha^i)$ are aggregate storage holdings held by financial intermediaries at t.

Definition 2: Equilibrium.

[37] We also do not constrain investment to be necessarily positive.

Let $\mathcal{S} = \{D_{t-1}, S^H_{t-1}, S^I_{t-1}, K_{t-1}, Z_{t-1}, \gamma_{t-1}, \varepsilon^z_t, \varepsilon^\gamma_t\}^\infty_{t=0}$ be the vector of state variables and shocks. Given a sequence of rates $\{r^D_t\}^\infty_{t=0}$, monetary policy rule and financial market rules $K(\mathcal{S}), \alpha^L(\mathcal{S}), S(\mathcal{S})$, let us define the optimal decisions of the representative household as $C^H(\mathcal{S}), D^H(\mathcal{S}), S^H(\mathcal{S})$.

An equilibrium is a sequence of rates $\{r^D_t\}^\infty_{t=0}$, and policy rules $C^H(\mathcal{S})$, $D^H(\mathcal{S})$, $S^H(\mathcal{S})$, $S^I(\mathcal{S})$, $K(\mathcal{S}), \alpha^L(\mathcal{S})$, such that:

- $C(\mathcal{S}), D^H(\mathcal{S}), S^H(\mathcal{S}), S^I(\mathcal{S}), K(\mathcal{S}), \alpha^L_t(\mathcal{S})$ are optimal given $\{r^D_t\}^\infty_{t=0}$
- Asset and goods markets clear at every period t

In equilibrium, we need to find a deposit rate which, conditional on exogenous variables and the financial sector equilibrium, is consistent with the household problem. We proceed by iterating on r^D_t, imposing the financial market equilibrium results. For a given deposit rate r^D_t, we find the law of motion for household wealth and consumption, use the Euler equation errors to update the deposit rate and repeat until convergence. A more detailed explanation of the algorithm used for our global solution method can be seen in Appendix A.

5.3 Calibration

To solve the model numerically, we need to specify the period utility function, the shape of the distribution of the *VaR* probabilities and calibrate the remaining parameters. Given the interaction between extensive and intensive margins, the mass of intermediaries in a given section of the distribution could have an important role in determining which of the two effects dominates. To highlight that the results described are not a consequence of this distribution, we assume that $G(\alpha^i)$ is uniform between $[0, \overline{\alpha}]$. For the utility function, we assume a standard CRRA representation.

$$u(C) = \frac{C^{1-\psi} - 1}{1 - \psi} \tag{46}$$

The calibration can be seen in Table 1. For the utility function parameters, risk aversion ψ, the subjective discount factor β, the TFP parameters ρ^z and σ_z we use

Table 1: Calibration of selected parameters

Parameter	Value	Description
ψ	4	Risk aversion parameter
β	0.96	Subjective discount factor
ρ^z	0.9	AR(1) parameter for TFP
σ_z	0.03	Standard deviation of TFP shock
μ_γ	0.023	Target spread over deposit rates
ρ_γ	0.816	Spread persistence
σ_γ	0.0128	Standard deviation of spread
$\frac{\chi}{1+\chi}$	0.41	Wholesale funding percentage
θ	0.35	Capital share of output
δ	0.1	Depreciation rate
ω	0.697	Equity of intermediaries
$\overline{\alpha}$	0.4961	Upper bound of distribution $G(\alpha^i)$
ζ	0.01	Idiosyncratic unproductive capital probability

standard values from the literature. Similarly for θ, the capital share of output, and for δ the depreciation rate of the capital stock. To calibrate the monetary policy parameters, we calculate the subsidy as the difference between the Effective Fed funds Rate and $1/\beta$, the long-run deposit rate. We then fit an AR(1) process to get the parameters used. The wholesale funding percentage used to calibrate χ was calculated from the time series mean of the cross-sectional asset-weighted average in Bankscope data[38] for the period 1993-2015. For the purpose of this calibration, wholesale funding was assumed to be all non-deposit liabilities of each financial intermediary. We calibrate $\overline{\alpha}$ to match the probability of default of the median *risky* intermediary when deposit rates are at the steady-state. Using FDIC data on failed banks, we find that the median age of failed banks in the US was around 20.5 years. The full sample distribution of ages at failure can be seen in Figure A4. We then calibrate $\overline{\alpha}$ to match a default probability of 5% for the median intermediary when $R_t^D = 1/\beta$. ω is chosen to fit leverage at the steady-state. Some of the intermediaries are leveraged and others are not, so we cannot use only Bankscope data (which contains mostly leveraged banks) to calibrate

[38]Bankscope contains a large panel of financial intermediaries' balance sheet data. See Appendix D.

leverage. According to the "broad measure" of Other Financial Institutions (OFIs) in the Global Shadow Banking Report (Financial Stability Board (2015)), non-levered intermediaries hold about 137 trillions of assets while banking assets are around 135 trillion. We use these figures to calculate an asset-weighted average of leverage of 7.3, which is reached by combining the Bankscope asset-weighted average leverage of 13.5 for 2015 and assuming a leverage of 1 for the OFIs. We target our calibration of ω so that the median *risky* intermediary matches this value. The size of the equity endowment ω and the volatility of aggregate shocks σ_z will also contribute to determine the financial sector reaction to changes in deposit rates. For that reason, we also conducted some comparative statics on both σ_z and ω to see how the model changes with those parameter calibrations. There is very little effect on the first moments of real variables such as output and consumption but there are important changes on equilibrium leverage and financial stability when we vary ω and/or σ_z. In general, the easier it is for riskier intermediaries to gain market share, the less stable will financial markets be. Increases in ω and decreases in σ_z both *worsen* financial stability *ceteris paribus*. Low volatility of the fundamental shocks σ_z will lead to lower financial stability since riskier intermediaries will find it easier to capture the market. More details can be found in Appendix F. The value of $\overline{\alpha}$ and the shape of its distribution will also matter for financial stability. Increasing $\overline{\alpha}$ leads to a less financially stable financial sector. We leave for future work to perform a (technically challenging) estimation of the model where the distributions of α or ω could potentially be backed out from the data and focus here on understanding the qualitative implications of the model.

5.4 Monetary policy and leverage

We now look at the impact of a positive subsidy shock, which we will refer to as an expansionary monetary policy shock or a decrease in the cost of funds. In Figure 8 we see the impact of a 100 basis points increase to the subsidy[39] in three different scenarios to illustrate the non-linear effects of monetary policy on financial stability. Impulse response functions are expressed as deviations from the respective scenario in

[39] Note that this translates into a lower reduction in the total cost of funds (see Figure 9). This is due to the fact that the cost of funds is a composite of deposits and wholesale funds, but also due to endogenous movements in the deposit rate.

the absence of the shock. This monetary policy loosening decreases the funding rate of the banks by 8 bp as can be seen in the left panel of Figure 9. Scenario 1 (blue line) features a low initial capital stock (corresponding to high equilibrium levels of the interest rate). Scenario 2 (red line) is for a larger capital stock (corresponding to a low level of equilibrium interest rate). Scenario 3 (black line) is at the risky steady-state[40]. As in Coeurdacier, Rey and Winant (2011) we define the risky steady-state as the steady-state in which there are no shocks but economic agents take into account the full stochastic structure of the model when they optimize (unlike in the deterministic steady-state where they expect no shocks).

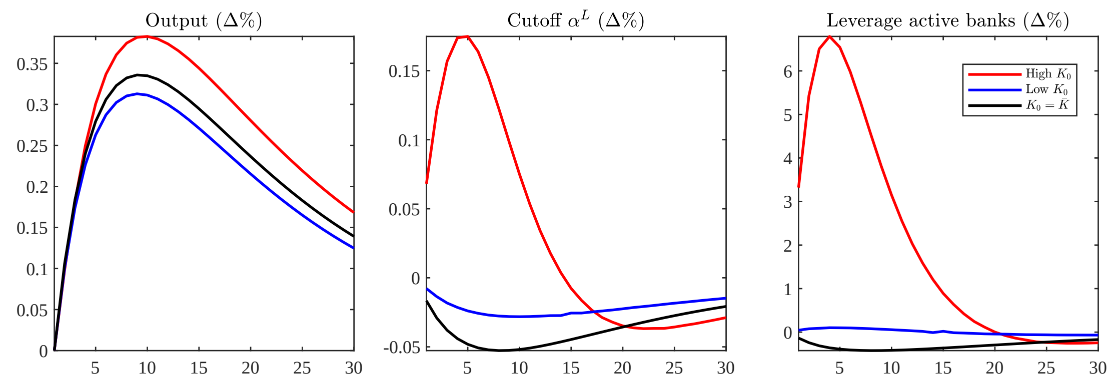

Figure 8: *Monetary policy shock of 100 basis points to* γ_t

We can easily relate the general equilibrium results to the partial equilibrium intuitions developed above. In the case of a low initial capital stock (associated with a high equilibrium funding rate), a positive monetary policy shock expands output, increases aggregate leverage and at the same time it reduces the cut-off α^L, due to the entry of less risk-taking intermediaries in deposit markets. We are in the "no trade-off zone of monetary policy" where a decrease in the interest rate increases investment and financial stability. In the case of a high initial capital stock (associated to a low funding cost for intermediaries), an expansionary shock has a larger positive effect on output and leverage but this time intermediaries at the margin choose not to lever

[40]These three scenarios were chosen to illustrate the parallel with the partial equilibrium setting, since the solution of the model is such that there is, ceteris paribus, a negative correlation between the initial capital stock and the funding rate.

up. In contrast, the most risk-taking intermediaries leverage significantly and financial stability is affected negatively.

This is a very different trade-off from the traditional Phillips curve which has been the benchmark model driving monetary policy analysis for many years. Aggregate economic variables such as consumption, wealth or capital behave smoothly as evidence in Figure A5, but the underlying change in financial structure supporting these macroeconomic outcomes can become less stable depending on the level of the interest rate.

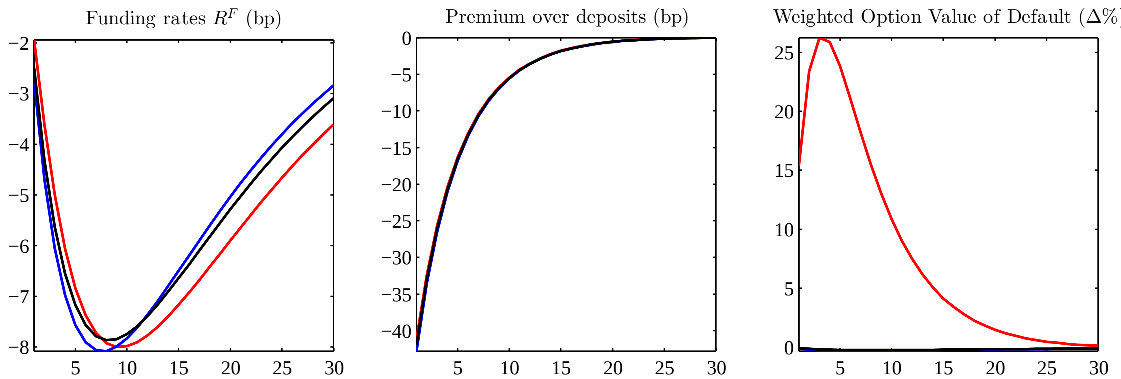

Figure 9: *Monetary policy shock of 100 basis points to γ_t: Financial variables*

As seen in Figure 9, the Weighted Option Value of Default also increases drastically with a monetary policy loosening when interest rates are low. The option value of default is defined as the difference between expected profits under limited liability and the (untruncated) cash flow. The larger this difference, the bigger the distortions coming from the presence of the limited liability and the worse for financial stability. Since the option value of default is intermediary-specific due to the heterogeneity of balance sheets, we construct an asset-weighted mean to illustrate the aggregate effect. When the interest rate is lower, the decrease in the cost of funds generates a very large increase due to the exit of safer intermediaries but also to the increase in leverage skewness in the cross-section. The impulse response functions for the alternative risk measures can be found in Figure A8 in Appendix C. They also illustrate the presence of a strong trade-off when interest rates are low. Finally, the premium over deposits *goes down* as monetary policy expands since the demand for deposits goes up and the expected return to risky capital goes down due to decreasing returns and the mispricing of aggregate risk. This is consistent with the literature documenting the effect of monetary policy on asset prices and risk premia (see Miranda-Agrippino and Rey, 2020). Scenario 2 has the characteristics of a "bad boom" in the terminology of Gorton and Ordoñez (2019).

5.5 Productivity driven leverage

Cycles in leverage can be driven by movements in the cost of funds, but also by changes in expected productivity. When leverage is driven by an increase in productivity then the ensuing leverage growth does not come at the cost of financial stability. There is a fundamental difference between a credit boom driven by a shift in supply (i.e. cheaper access to funds) and a boom driven by demand for credit (i.e. better investment opportunities). Productivity shocks in our framework are an example of the latter as they forecast larger productivity in the future. In general equilibrium supply and demand of credit are interdependent so this distinction is simply to clarify the intuition and to relate it to the original shock leading to credit growth. We now look at a shock to productivity. In Figure 10 we see the impact of a one standard deviation positive productivity shock in the same 3 scenarios as before.

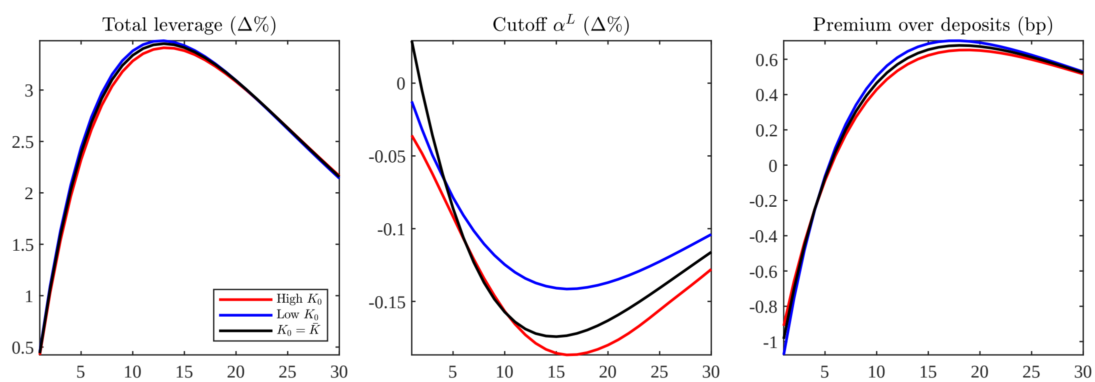

Figure 10: *Shock to exogenous productivity*

The effects are similar irrespective of the position in the state space and the level of interest rates at the time of the shock. Total leverage goes up due to increased investment opportunities and is hump-shaped, as can be seen in the first panel. The hump-shape is due to the initial pressure of credit demand which requires higher deposit rates to clear the market. After impact, household wealth accumulates and deposit rates start to fall, leading to a hump-shaped response of credit and investment as the positive productivity shock fades out. This effect can be also seen on the premium over deposits (right panel). On impact, there is a larger rise of deposit rates than expected returns, despite the better investment opportunities coming from higher expected productivity. The effect on the premium is however very small (1bp decrease on impact), only a small fraction of the effect seen after a monetary policy shock (40bp). In the middle panel, we also see that financial stability overall slightly improves in all scenarios, apart from a short-lived marginal uptick on impact in the middle scenario. Again these are small effects, indicating that productivity driven leverage booms are not a concern for financial stability in the same way that credit supply driven ones are. As Krishnamurthy and Muir (2017) show, credit booms accompanied by the tightening of spreads can predict financial crises, while those without such a tightening do not. We are able to rationalize this fact through the cross-sectional composition of the financial sector and the difference between productivity driven and credit supply driven leverage. The positive productivity shock leads to a "good boom".

6 Costly intermediary default

We now consider the case of costly intermediary default. Leveraged intermediaries in risky financial markets will default on depositors if the realisation of the productivity shock is low enough[41]. This requires intervention by the government to pay for deposit insurance, which is now less benign than previously assumed as there is a deadweight loss[42]. We assume that capital held by defaulting intermediaries suffers a proportional productivity loss $\overline{\Delta}$ relative to the productivity of capital held by non-defaulting intermediaries. This loss can arise from (real) bankruptcy costs or some degree of inalienability in investment projects. The main assumption is that these costs are proportional to the output of the respective capital shares. Let μ_t^d be the share of capital held by defaulting intermediaries. The aggregate productivity loss $\Delta_t = \mu_t^d \overline{\Delta}$

[41]We note that even relatively small negative productivity shocks in our model can lead to large crises if many assets are in the hands of the most risk-taking intermediaries.

[42]As before, deposit guarantees will be financed by lump sum taxation of households. The welfare analysis of our setup is left for future work.

is then increasing in the share of capital held by defaulting intermediaries. Note that the productivity of capital held by healthy intermediaries is unaffected at t, so the impact on aggregate productivity is coming only from cross-sectional differences between defaulting and non-defaulting intermediaries. The extent of the aggregate "loss given default" is therefore endogenous in our model and increasing with the severity of the banking crisis.

We also consider the possibility that this disruption spreads to the entire financial market in the following periods by affecting productivity of *all* intermediaries in future periods by Δ_t, which reflects the endogenous severity of the financial crisis. The loss of aggregative productivity is then intermediary-specific during default, but it can affect the whole economy moving forward (the allocative process of the whole economy is impaired). When it happens we call this the *crisis* state. We model the persistence of the crisis state through a Poisson process, with a constant probability p of exiting the crisis at each period. Depending on the process, variable ξ_t takes the value of one if the crisis carries on to the next period or zero if it does not. This reflects for example the (unmodelled) capacity of the state to intervene and solve the banking crisis quickly or not[43]. Our specification nests both the case of costless default ($\overline{\Delta} = 0$) and the case where there is no disruption of financial markets in subsequent periods ($p = 1$). We have:

$$\mu_t^d = \frac{\int k_{it} \ \mathbb{1}_{(\pi^i < 0)} \ dG(\alpha^i)}{K_t}$$
$$\Delta_t = \xi_{t-1} \max(\mu_{t-1}^d \ \overline{\Delta}, \Delta_{t-1})$$

where the indicator function takes the value of 1 if intermediaries of type i default or 0 if not. If there are also defaults during a crisis state, then the max operator ensures that the largest penalty applies going forward. Whenever the economy is in crisis,

[43]The length and severity of financial crises are heterogeneous in the data. States have shown a very uneven ability to clean financial sectors balance sheets and dispose of non performing loans. Politics and economics are closely intertwined in that process. Our assumptions on real default costs are similar to the sovereign default literature, i.e. Arellano (2008) who calibrates output costs due to default and the probability of re-entry in capital markets (like we do with the Poisson process) from the data.

productivity for all financial intermediaries is scaled down by a factor μ_t^d proportional to the percentage of total capital held by defaulting intermediaries. ξ_{t-1} is known to agents when they make their investment decisions at period $t-1$, so the uncertainty on the returns on their capital investment is only on the realization of the exogenous productivity process[44]. This timing assumption allows us to keep tractability as the main difference in the financial sector block is that now $Z_{t+1}^e = (1-\Delta_t) Z_t^{\rho^Z}$. Since both Δ_t and Z_t are state variables, we can still solve for the financial sector equilibrium as before.

This set up is tractable and allows us to parameterize crises of different severity and length. Reinhart and Rogoff (2009*a*) present a classic description of the characteristics of crises across history, and evidence that crises associated with banking crises are more severe. Borio et al. (2016) and Laeven and Valencia (2012) present empirical evidence showing that there can be substantial and long lasting productivity drops after financial crises. To calibrate these parameters we refer to the database of Laeven and Valencia (2012), setting $p = 0.5$ to target an average crisis length of 2 years as in the data, and $\overline{\Delta} = 0.115$ implying a maximal efficiency loss of 11.5% per year[45].

6.1 Productivity shocks and financial crises

In this section we study the impact of a financial crisis with costly default on the path of the economy, following a productivity shock. Figure 11 shows the impact of a large productivity shock in 3 possible scenarios[46].

In scenario 1 (red line) the economy at the risky steady-state is hit at period t by the largest possible shock that does not trigger any default. In scenarios 2 (blue line) and 3 (black line) the economy is hit with the smallest shock such that all *levered* intermediaries default. The difference between scenarios 2 and 3 is the length of the crisis. Scenario 2 is the short crisis scenario, where the crisis only carries on to the next period, $\xi_1 = 1$. Scenario 3 is the "unlucky" scenario, where the crisis carries on

[44]There is still uncertainty on asset returns if the intermediary defaults but this is not considered in the intermediary problem due to limited liability truncating the profit functions at zero in those states.

[45]In the database of Laeven and Valencia (2012), the average cumulative output loss is 23% over the length of the crisis, which is on average two years.

[46]Impulse response functions are expressed in basis points deviations for rates or otherwise in percent deviations from the risky-steady state

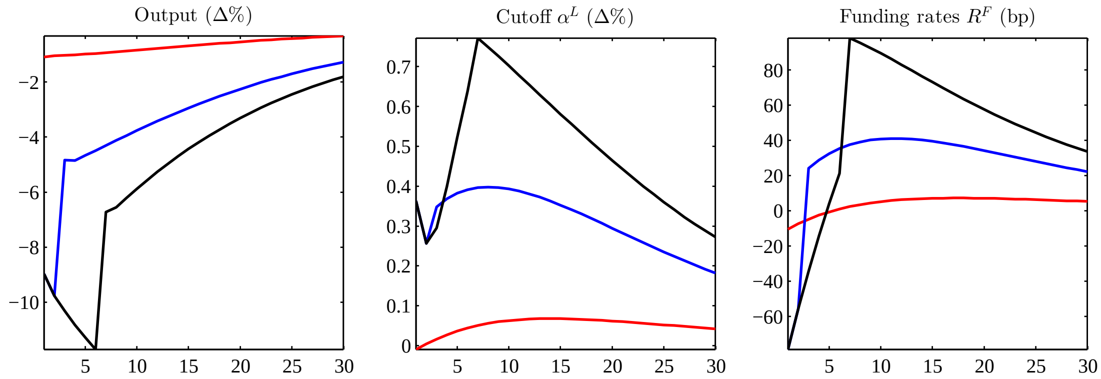

Figure 11: *Large shock to exogenous productivity*

for an additional 5 periods, possibly because of (unmodelled) policy mistakes: $\xi_s = 1$ for $t = 1$ to $t = 6$. The length of the crisis is unknown beforehand to the agents in the economy, although as mentioned before they observe the value of ξ_t when they make their investment decisions at t. Not surprisingly, when the crisis hits there is a large decline in output. As expected productivity is low, only the intermediaries with the looser *VaR* constraints can operate. There is a strong fall in deposit demand due to the low expected productivity, which severely tightens the constraint. In equilibrium the fall in deposit demand generates a fall in funding costs due to decreased deposit rates. For the cases with defaults, one can observe a small initial decrease in the cut-off after an initial jump. This is because on impact the economy jumps to the trade-off region. As interest rates start rising from that point onward, the economy travels through the U-shape with the cut-off falling initially and then increasing as interest rates rise. The length of the crisis also has very interesting dynamic effects on wealth. Given that households expect to exit the crisis state with probability p, when exit fails to materialize in Scenario 3 they are running down their wealth and their consumption dips down (see Figure A6). As wealth falls, deposit rates and funding costs (see Figure 11) grow as it becomes more costly for the household to save and fund bank leverage. When eventually the economy exits the crisis state, household wealth is low and demand for leverage jumps, leading to a jump in funding rates to compensate households for decreased consumption today. This leads also to a higher risk premium as expected return to capital jumps up. Total leverage and investment, which had seen severe contractions start to go up again (see Figure A6). This effect is also present with a short crisis, but is particularly stark for the longer crisis.

7 Conclusion

This paper develops a novel framework for modelling a financial sector with heterogeneous financial intermediaries and aggregate risk. The heterogeneity in the *VaR* constraints coupled with limited liability generates endogenous time variation in leverage, risk-shifting and financial stability. The interaction between the intensive and the extensive margins of investment creates a rich set of non-linear dynamics where the level of interest rates plays a key role. There is a risk-taking channel of monetary policy. When interest rates are high, a monetary expansion increases both the intensive margin and the extensive margin. The monetary authority is able to stimulate the economy, while at the same time increasing financial stability. When interest rates are already low, a further reduction can lead to large increases in leverage by the most risk-taking institutions, pricing out previously active intermediaries, due to decreasing aggregate returns to capital. Importantly, the intermediaries which decrease their balance sheet size have lower probabilities of default than those that remain levered, leading to an increase in systemic risk. Our model, unlike the existing literature, generates a trade-off between economic activity and financial stability depending on the level of the interest rate. During booms driven by low funding costs and increased credit supply, risk premia are low as there is a lot of risk-shifting by the most risk-taking intermediaries in the economy. Booms driven by positive productivity shocks do not lead to an increase in financial instability nor to such low levels of risk premia. Because our framework has heterogeneity at its heart, it allows us to make use of cross-sectional data on intermediary balance sheets. We derive novel implications linking the times series of the concentration of leverage and monetary policy which are strikingly borne out in the data. We believe we are the first paper to link changes in the cross-sectional distribution of leverage, macroeconomic developments and fluctuations in financial stability. We show that similar macroeconomic outcomes can be supported by very different underlying financial structures. This has important implications for the transmission of monetary policy, the effect of fundamental volatility, of financial deregulation or of capital flows and the sensitivity of the economy to interest rate movements. For example, in our model, looser monetary policy decreases risk premia as in the data and we can explain the volatility paradox.

A major advantage of our framework is that our financial block is easy to embed in a standard dynamic stochastic general equilibrium framework. We plan to extend our model to environments with sticky prices and a more complex portfolio choice on the bank side as well as to study boom and bust cycles in emerging markets. We also plan to apply it to explain the dynamics of the real estate market. The model could also be calibrated to fit a distribution of financial intermediaries characteristics. One could in principle back out the distribution of α^i from leverage data and allow for a distribution of intermediary-specific equity ω^i (see Coimbra, Kim and Rey (2021)). That said, allowing for endogenous dynamics in equity would require the introduction of an additional state-variable in the financial sector problem which would make the solution more computationally intensive.[47] We leave these issues and the welfare implications of our model for future research.

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[47] And having together time-varying and intermediary-specific equity would require an infinitely dimensional state-space without additional assumptions.

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Tables and Figures

Table A1: Baseline regressions

	(1) $Lev_{i,t}$	(2) $Lev_{i,t}$	(3) $Lev_{i,t}$	(4) $Lev_{i,t}$	(5) $Lev_{i,t}$	(6) $Lev_{i,t}$	(7) $\Delta Lev_{i,t}$
$Lev_{i,t-1}$	0.454***	0.454***	0.455***	0.464***	0.463***	0.490***	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
$Top5_{i,t}$	15.24***	15.34***					3.552**
	(0.000)	(0.000)					(0.004)
$Top5_{i,t} \times FF_t$	-2.251***	-2.252***					
	(0.000)	(0.000)					
FF_t	0.0467						
	(0.466)						
$Top5^{AW}_{i,t}$			27.87***				
			(0.000)				
$Top5^{AW}_{i,t} \times FF_t$			-1.917***				
			(0.000)				
$Top10_{i,t}$				8.361***			
				(0.000)			
$Top10_{i,t} \times FF_t$				-0.802***			
				(0.001)			
$Top10^{AW}_{i,t}$					9.993***		
					(0.000)		
$Top10^{AW}_{i,t} \times FF_t$					-0.794***		
					(0.000)		
$Lev_{i,t-1} \times FF_t$						-0.0173***	
						(0.000)	
$Top5_{i,t} \times \Delta FF_t$							-1.164**
							(0.003)
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time Effects	No	Yes	Yes	Yes	Yes	Yes	Yes
R^2 (within)	0.252	0.254	0.292	0.248	0.246	0.234	0.007
R^2 (overall)	0.772	0.772	0.674	0.762	0.736	0.758	0.020
N	5325	5325	5325	5325	5325	5325	5325

p-values in parentheses: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A2: Equity and Asset Regressions

	(1) $Lev_{i,t}$	(2) $Lev_{i,t}$	(3) $Lev_{i,t}$	(4) $\Delta Lev_{i,t}$	(5) $Lev_{i,t}$	(6) $Lev_{i,t}$	(7) $Lev_{i,t}$
$Lev_{i,t-1}$	0.485***	0.485***	0.480***		0.454***	0.484***	0.489***
	(0.000)	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)
$Top5^{Eq}_{i,t}$	-1.706	-1.691		-1.010	-1.096		
	(0.109)	(0.112)		(0.377)	(0.296)		
$Top5^{Eq}_{i,t} \times FF_t$	0.329	0.330			0.226		
	(0.255)	(0.253)			(0.427)		
FF_t	-0.0370						
	(0.573)						
$\log Eq_{i,t-1}$			-0.543			-0.631*	
			(0.070)			(0.035)	
$\log Eq_{i,t-1} \times FF_t$			0.0604			0.0647	
			(0.103)			(0.080)	
$Top5^{Eq}_{i,t} \times \Delta FF_t$				0.277			
				(0.435)			
$Top5_{i,t}$					15.27***		
					(0.000)		
$Top5_{i,t} \times FF_t$					-2.249***		
					(0.000)		
$Lev_{i,t-1} \times FF_t$						-0.0178***	-0.0183***
						(0.000)	(0.000)
$\log Assets_{i,t-1}$							0.264
							(0.383)
$\log Assets_{i,t-1} \times FF_t$							0.0433
							(0.231)
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time Effects	No	Yes	Yes	Yes	Yes	Yes	Yes
R^2 (within)	0.228	0.230	0.231	0.00415	0.255	0.235	0.234
R^2 (overall)	0.763	0.763	0.755	0.00228	0.771	0.746	0.756
N	5325	5325	5325	5325	5325	5325	5325

p-values in parentheses: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

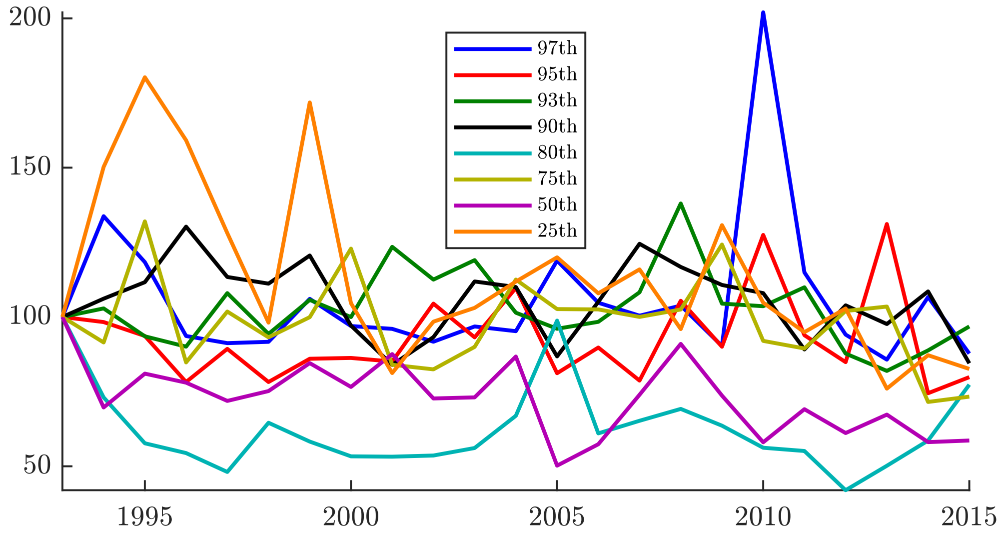

Figure A1: Leverage index by equity quantiles (indexed to 100 for base year 1993)

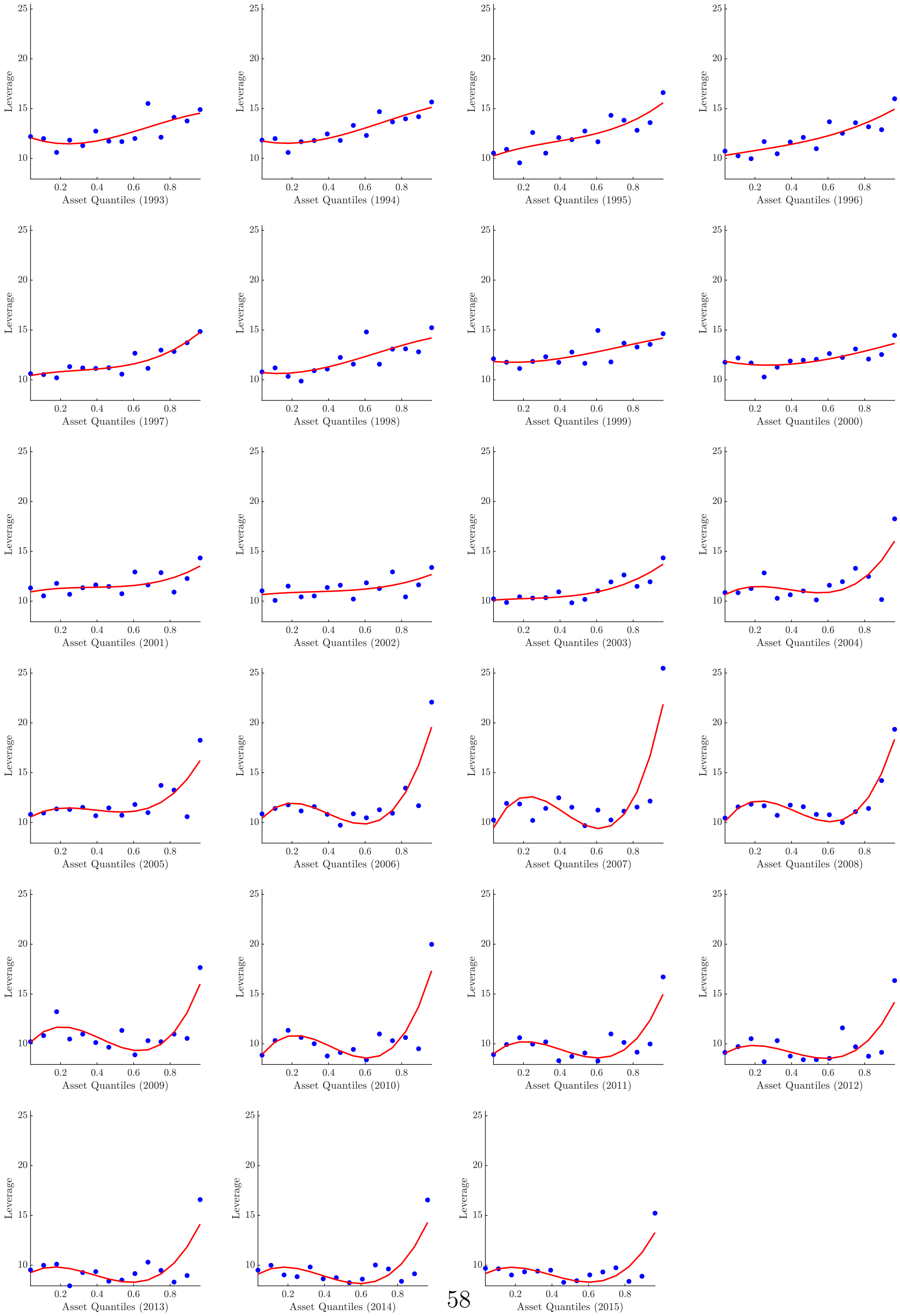

Figure A2: Binned scatter plot of leverage on asset quantiles by year.
Binned scatter plots with intermediaries grouped in the x-axis into 14 bins by balance sheet size. Red lines are fitted cubic polynomials. Each bin contains an average of 30 observations.

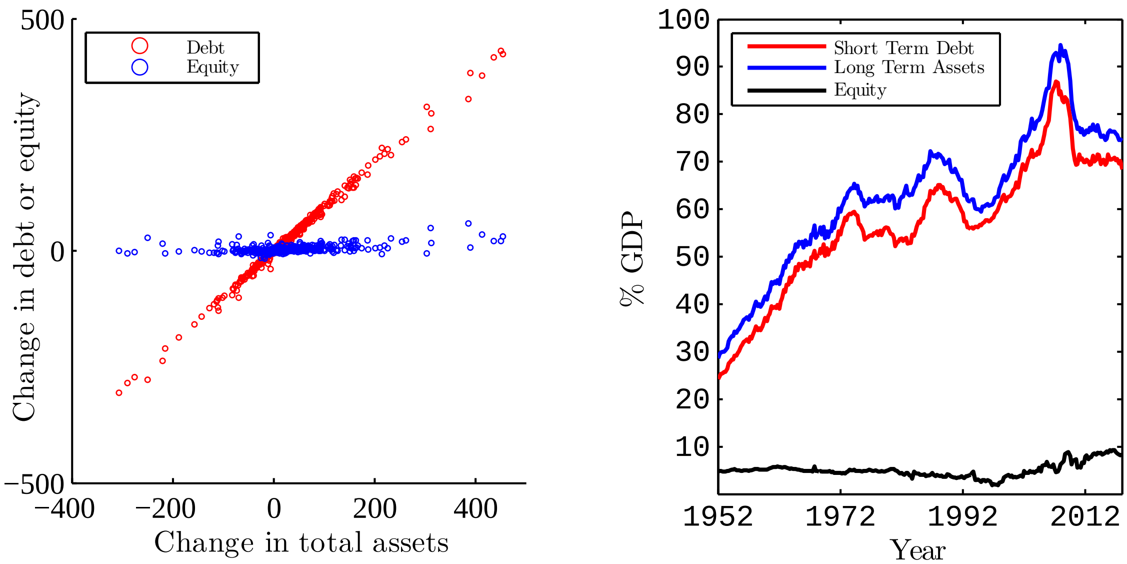

Figure A3: Left panel: Yearly changes in total asset against yearly changes in equity or debt from 1993 to 2015. Billions of USD. Source: Bankscope. Right panel: Bank short-term debt, long-term assets and equity as a percentage of US GDP. Data constructed as in Krishnamurthy and Vissing-Jorgensen (2015)

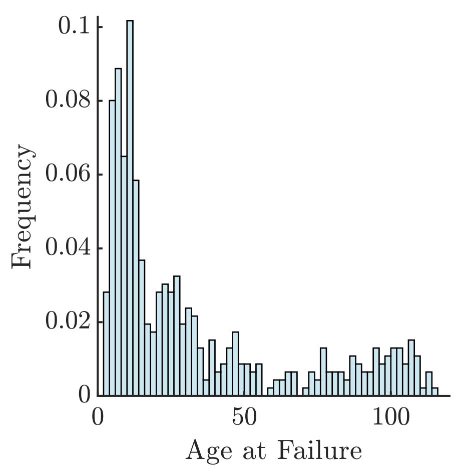

Figure A4: Histogram of age of banks at closing date (in years).
Data for failures in the US since October, 2000. Source: FDIC.

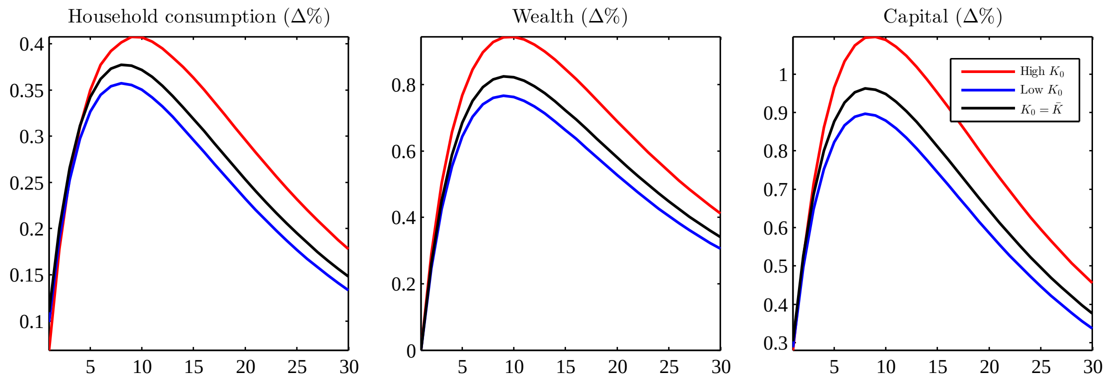

Figure A5: *Monetary policy shock of 100 basis points to γ_t: Real variables*

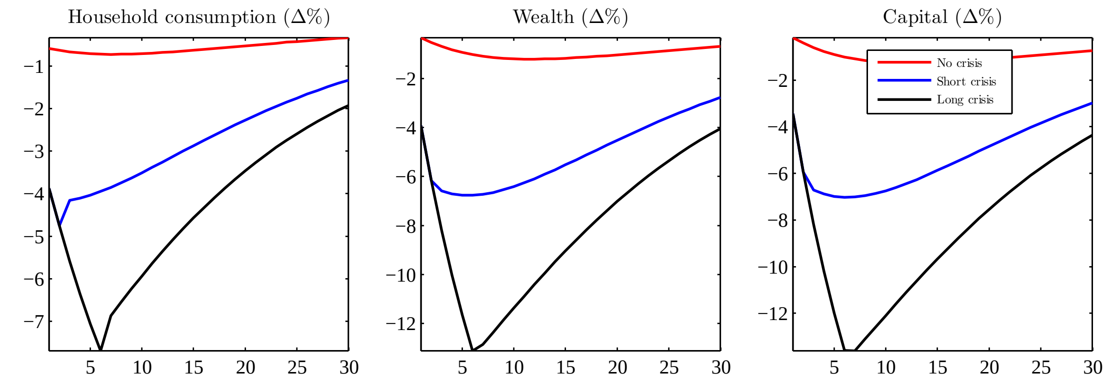

Figure A6: *Large shock to exogenous productivity: Real variables*

Appendix A. Numerical solution method

The solution method is composed of two main blocks. The first block solves the partial equilibrium problem for a grid of points for variables r^F and Z^e. We discretize the state space using 100 nodes for Z^e and 200 for r^F. Given funding costs r^F and expected productivity Z^e we can solve jointly for equations (26) and (28), plugging in equation (21) in the latter. We also use the property that levered intermediaries never invest in storage. This gives us policy functions $K^*(r^F, Z^e)$ and $\alpha^{L,*}(r^F, Z^e)$.

The second block is the recursive one. First we define the household savings problem as a function of disposable wealth Ω_t, productivity $\tilde{Z}_t$, efficiency adjustment Δ_t and monetary policy γ_t.

$$\Omega_t = (1-\theta)Y_t - T_t + D^H_{t-1} + S^H_{t-1}$$

The procedure entails the following steps

1. Discretize the state space $\mathcal{S}$ for the variables $(\Omega, Z, \Delta, \gamma)$. The process for Z and γ are approximated using a Tauchen and Hussey (1991) quadrature procedure with 11 and 7 nodes respectively. The state space for the variable Ω is discretized using 500 nodes and we use 10 for Δ.

2. Iterate on prices r^D and policy function $C^*(\mathcal{S})$ starting with an initial guess $r^D(\mathcal{S})$ for deposit prices and $C^*(\mathcal{S})$. For every point $\mathcal{S}_j \in \mathcal{S}$:

   (a) Using the state vector and r^D_j, calculate r^F_j and Z^e_j.

   (b) Solve for (K_j, α^L_j) using $K^*(r^F_j, Z^e_j)$ and $\alpha^{L,*}(r^F_j, Z^e_j)$. Back out deposit supply D_j from the balance sheet equations.

   (c) Plug D_j in the budget constraint of the agent. Together with $C_j = C^*(\mathcal{S}_j)$ this pins down S^H_j.

   (d) Update deposit prices to $\tilde{r}^D_j$ and policy functions to $\tilde{C}^*_j$ using the optimality conditions and numerical integration to calculate expectations.

   (e) Check for convergence. If $||(\tilde{r}^D_j - r^D_j)|| + ||(\tilde{C}^*_j) - C^*_j||$ is smaller than a threshold value stop. Else, go back to (a) and repeat.

To numerically integrate intermediary variables, Gauss-Legendre quadrature using 51 points is used. To calculate expectations of future net disposable wealth, we also need to calculate taxes conditional on future shocks. For a given productivity draw $Z'|Z_j$ we identify the threshold intermediary for which no bailout is needed: $(R^K k_i - R^D d_i) = \omega$. We can then calculate the amount T_t of taxes required by numerical integration.

Appendix B. Proof of Proposition 3.1

When $\mathbb{E}[R^K_{i,t+1}] \geq 1$, participating intermediary i will either lever up to its Value-at-Risk constraint: $d_{it} = \overline{d}^i_t$, or not raise deposits at all : $d_{it} = 0$.

Given the option value of default and the condition $\mathbb{E}[R^K_{i,t+1}] \geq 1$, participating intermediaries will not invest in storage. The Value-at-Risk constraint bounds the maximum level of leverage of intermediary i, therefore $d_{it} \in [0, \overline{d}^i_t]$. The expected profits of intermediary i as a function of deposits are:

$$\pi^i_t(d_{it}) = (1-\zeta)\int_{\varepsilon^i_t(d_{it})}^{\infty} \left[R^K_{t+1}(\omega + d^i_t) - R^D_t d^i_t\right] dF(\varepsilon) \tag{47}$$

where ε^i_t is the max of 0 (the lower bound of the support for ε) and the shock for which profits are zero).

$$\varepsilon^i_t(d^i_t) = \max\left(0, \frac{\frac{R^D_t d^i_t}{\omega + d^i_t} - 1 + \delta}{\theta Z^{\rho^Z} K^{\theta-1}_t}\right) \tag{48}$$

Taking derivatives:

$$\frac{\partial \pi^i_t}{\partial d^i_t} = (1-\zeta)\int_{\varepsilon^i_t(d^i_t)}^{\infty} \left(R^K_{t+1}(\varepsilon) - R^D_t\right) dF(\varepsilon) - \pi^i_t(\varepsilon^i)\frac{\partial \varepsilon^i}{\partial d^i_t} \tag{49}$$

Lemma 1 *Given equations (47) and (48), then* $\pi^i_t(\varepsilon^i_t)\frac{\partial \varepsilon^i}{\partial d^i_t} = 0$

For any $d_{it} \geq \frac{\omega(1-\delta)}{R^D_t - 1 + \delta}$, then $\pi^i_t(\varepsilon^i_t) = 0$ by definition of ε^i_t. For $d^i_t < \frac{\omega(1-\delta)}{R^D_t - 1 + \delta}$, then $e^i = 0$ and $\frac{\partial \varepsilon^i}{\partial d^i_t} = 0$ due to the max operator.

We have as first and second derivatives:

$$\frac{\partial \pi_t^i}{\partial d_{it}} = (1-\zeta) \int_{\varepsilon_t^i(d_t^i)}^{\infty} \left(R_{t+1}^K(\varepsilon) - R_t^D \right) dF(\varepsilon)$$
$$\frac{\partial^2 \pi_t^i}{\partial d_{i,t}^2} = -(1-\zeta) \left[R_{t+1}^K(\varepsilon(d_{it})) - R_t^D \right] \frac{\partial \varepsilon_t^i}{\partial d_{it}} \tag{50}$$

Given the monotonicity of $R_{t+1}^K(\varepsilon)$, then $\forall \tilde{d}$ such that $\left. \frac{\partial \pi_t^i}{\partial d_{it}} \right|_{\tilde{d}} = 0$, it follows that $R_{t+1}^K(\varepsilon_t^i(\tilde{d})) - R_t^D < 0$ or all elements in the integral are non-negative and it cannot be zero. Since $\frac{\partial \varepsilon_t^i}{\partial d_{it}} > 0$, then $\left. \frac{\partial^2 \pi_t^i}{\partial d_{it}^2} \right|_{\tilde{d}} > 0$ by equation (50). If $\tilde{d}$ exists, it must be a minimum and we therefore conclude that the maximum must be at the bounds: $d_{it} = \arg\max \Big(\pi_t(0), \pi_t^i(\overline{d}_t^i) \Big)$.

Appendix C. Alternative Measures of Financial Stability

We present three alternative measures of financial stability. M^3 is the *asset-weighted mean of active* α^i. We have that $M_t^3 = \int_{\alpha_t^L}^{\overline{\alpha}} \alpha^i \frac{k_{it}}{K_t^L} dG(\alpha^i)$, where K_t^L is the total asset holdings of leveraged intermediaries. This measure has the advantage of not only capturing the extensive margin effect but also capturing the effect of skewness on aggregate financial stability. A financial sector with the same cut-off α_t^L but with a more skewed distribution of leverage will on aggregate be more risky, as a larger share of the capital is held by more risk-taking intermediaries.

We also explore a fourth measure of financial stability M^4: the *probability that a fraction* κ *of the capital* K_t^L *is held by distressed intermediaries in the next period.* M_t^4 is the solution to the equation $\int_{M_t^4}^{\overline{\alpha}} k_{it} dG(\alpha^i) = \kappa K_t^L$. This measure would be equivalent to the baseline measure M^1 if we set $\kappa = 1$, so it can be seen as a generalization of the first measure. Setting this fraction to a lower value captures some of the skewness effects mentioned. We implement this measure with a fraction arbitrarily set at $\kappa = 0.5$, so the probability that half of the capital is held by distressed intermediaries in the next period.

Finally we also calculate a fifth measure, M^5: *the expected share of capital held by defaulting intermediaries at* $t+1$. This measure relates to the costly default described in Section 6. If the deadweight loss is proportional to the share of capital held by defaulting intermediaries, then this measure gives us a sense of the expected efficiency costs of decreasing financial stability. Figure A7 shows these measures as a function of the interest rate in partial equilibrium. All of them show a significant adverse effect of an interest rate decrease on financial stability when interest rates are low. In contrast financial stability does not worsen with a decrease in the interest rate when the level of the interest rate is high. Figure A8 shows the effect of a monetary loosening on those three measures.

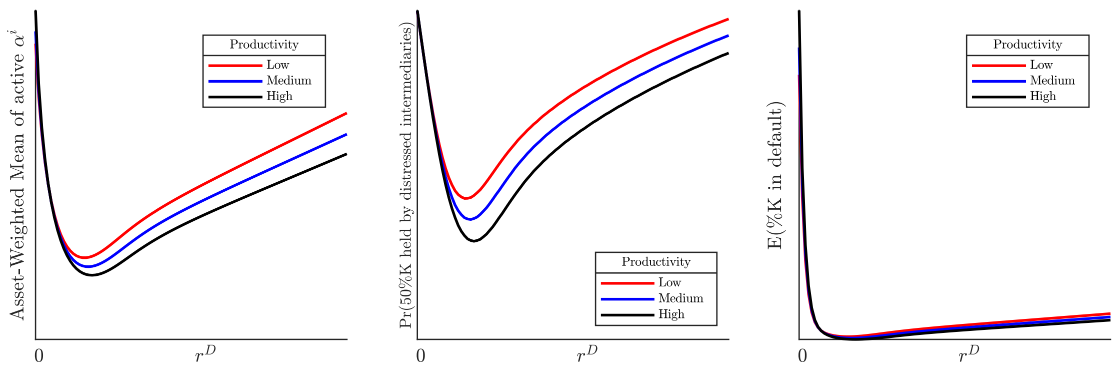

Figure A7: *Alternative measures of financial stability and interest rate*

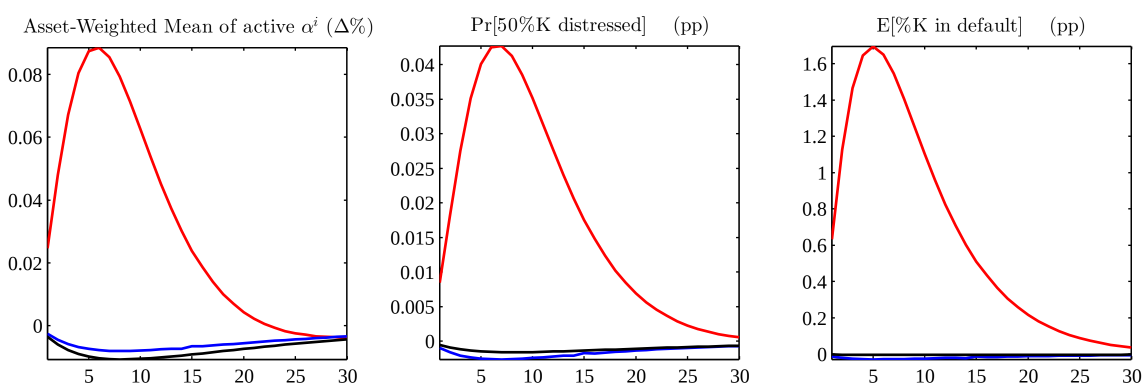

Figure A8: *Monetary policy shock of 100 basis points to* γ_t*: Alternative measures*

Appendix D. Data Description

Intermediary balance sheet data are from the Bankscope database at annual frequency and consolidation level C1 (mother company integrating the statements of its controlled subsidiaries or branches with no unconsolidated companion). Bank return data are from Datastream. Market returns were calculated using the MSCI World Index data available from Bloomberg. The Effective Federal Funds Rate and the CPI are from the Federal Reserve Economic Data. Data on the age of failed banks at closing date are from the FDIC.

From our sample of intermediary balance sheets we remove institutions labeled as "central banks", "specialized governmental credit institutions", "Islamic banks", "multi-lateral governmental banks" and "clearing & custody institutions". We include institutions from the following countries: United States, France, Germany, Canada, Japan, Netherlands, United Kingdom, Italy, Belgium, Switzerland, Spain, Australia, Brazil, Ireland, Norway, Sweden, Russia, Hungary, Portugal, Iceland, New Zealand, South Korea, Denmark and Hong Kong.

The leverage ratio is defined as the ratio of total assets to total equity, here defined as common equity. We drop negative equity from the dataset, and institutions with assets worth less than 1 million USD. We also remove institutions that have leverage larger than 1000 at least once across the sample.

For the leverage series, we consider both unweighted and weighted leverage ratios. For the weighted series we use total assets as weights. We checked using total equity as weights and results are qualitatively unchanged. We define asset-weighted leverage as $l_{it}\frac{a_{it}}{1/N_t \sum_{j=1}^{N_t} a_{jt}}$ where a_{it} and l_{it} are, respectively, assets and leverage of intermediary i at time t. N_t is the number of cross-sectional observations for period t.

Year	Assets		Equity		Leverage		#Obs
	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.	
1993	12419	39962	737	1930	14.71	13.77	276
1994	13725	44263	825	2179	14.81	10.95	329
1995	14638	42323	898	2200	14.33	11.16	349
1996	14278	35918	912	2109	14.82	16.26	364
1997	16778	42793	1073	2469	14.57	16.96	369
1998	18661	47154	1290	3158	13.71	10.69	392
1999	21478	64999	1526	4632	14.30	13.26	437
2000	24176	77576	1768	5599	14.04	15.11	456
2001	27648	87629	2079	6843	14.00	17.40	446
2002	28693	91765	2224	7328	14.09	25.57	458
2003	32012	103752	2414	8030	14.52	28.92	454
2004	44022	149190	3160	10731	14.58	17.96	444
2005	46877	152941	3287	10794	16.28	27.52	468
2006	56112	193439	3697	12586	15.69	24.76	422
2007	71684	245968	4458	14634	15.23	19.00	396
2008	67652	258317	4359	16032	15.59	27.13	374
2009	63755	240442	4872	17701	14.54	17.88	401
2010	66402	232329	5191	17920	12.90	11.93	448
2011	64599	235938	5318	18441	12.96	20.23	461
2012	65011	238222	5570	19317	12.51	21.56	467
2013	69559	249110	6468	22352	10.69	7.47	469
2014	70429	245651	6734	23223	10.58	7.71	475
2015	67587	228692	6923	23808	10.43	7.31	439

Table A3: Descriptive cross-sectional statistics by period (unweighted). Source: Bankscope.

Appendix E. Interbank market

In this appendix, we present a version of the baseline model where intermediaries can supply funds to each other through deposits. The main difference in the financial intermediary problem, is that inactive intermediaries will optimally choose to deposit their net worth, thus supplying funds to leveraged banks. These deposits are also guaranteed by the government and therefore the same asset as household deposits from the point of view of the borrowing bank.

Whenever $R_t^D > 1$, storage is dominated by deposits and will never be used. Inactive intermediaries will also optimally prefer to hold deposits over shares of the capital stock. Since intermediaries are risk-neutral, the presence of an option value of default implies that in equilibrium $R_t^D > E(R_{t+1}^K)$. Since inactive intermediaries will not be able to exploit the option value of default, they strictly prefer deposits over shares of the capital stock, implying $\alpha_t^N = \alpha^L$ for all t. The balance sheet of an intermediary i that chooses to lend its net worth is then:

Assets	Liabilities
$-d_{it}$	ω_{it}

where to maintain consistency in notation, deposits held as assets are noted as negative d_{it}. The intermediary program is as before:

$$
\begin{aligned}
V_{it} = & \max \ \mathbb{E}_t(c_{i,t+1}) \\
\text{s.t.} \ & \Pr(\pi_{i,t+1} < \omega_{it}) \leq \alpha^i \\
& k_{it} + s_{it} = \omega_{it} + d_{it} \\
& c_{i,t+1} = \max(0, \pi_{i,t+1}) \\
& \pi_{i,t+1} = R^K_{i,t+1} k_{it} + s_{it} - R^D_t d_{it}
\end{aligned}
$$

Since borrowing to deposit is revenue neutral, it follows that Proposition 3.1 again holds in this case. Each intermediary will choose to leverage up to its VaR constraint or not raise deposits at all. Writing the value functions under this case we have

$$
\begin{aligned}
V^L_{it} &= \mathbb{E}_t \left[\max\left(0, R^K_{i,t+1} k_{it} - R^D_t d_{it}\right)\right] \\
V^N_{it} &= R^D_t \omega \\
V^O_{it} &= \omega
\end{aligned}
$$

The deposit market clearing equation is as before:

$$
D_t = \int d_{it} dG(\alpha^i) = D^H_t
$$

With the difference that now D_t is the *net* borrowing from the financial sector as a whole. The market clearing is $D_t = D^H_t$, where D^H_t are total household deposits. We also define $D^L_t = \int_{\alpha^L_t}^{\overline{\alpha}} d_{it} dG(\alpha^i)$ as the total deposit liabilities in levered intermediaries. Equation (43) then becomes:

$$
F_t = \frac{D^L_t}{1 + \chi}
$$

The rest of the equations of the model are exactly the same, but underlying them are a few key differences. All capital is now held by levered intermediaries, which implies that no fraction of the capital stock is ever free from potential distress at $t + 1$. Moreover, the extensive margin now also affects the deposit supply. The more intermediaries drop out from levered markets, the larger is aggregate deposit supply (ceteris paribus). Partial equilibrium results are very similar to the ones without the interbank market, as can be seen in Figure A9. Note that for the aggregate capital stock supply curve, the two models are almost indistinguishable. The main difference in partial equilibrium

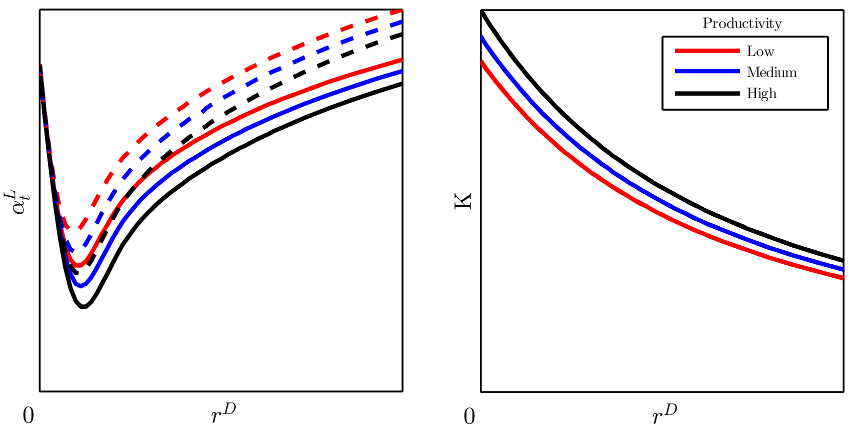

Figure A9: *Cut-off level α_t^L and aggregate capital stock as a function of deposit rates r_t^D in the model with an interbank market (full lines). For comparison, the baseline model is also plotted (dotted lines).*

is that for a given interest rate, the cut-off is now lower. Non-active intermediaries no longer invest directly in the capital stock. Had leverage and the cut-off remained the same the capital stock would be smaller and returns higher. This leads to both higher leverage from intermediaries above the cut-off (intensive margin) and a lower cut-off (extensive margin).

In general equilibrium, the main results are extremely similar to our baseline model as can be seen in Figures A11 and A12 . The main difference is the behavior of the cut-off where the baseline model has some additional amplification, particularly away from the steady-state.

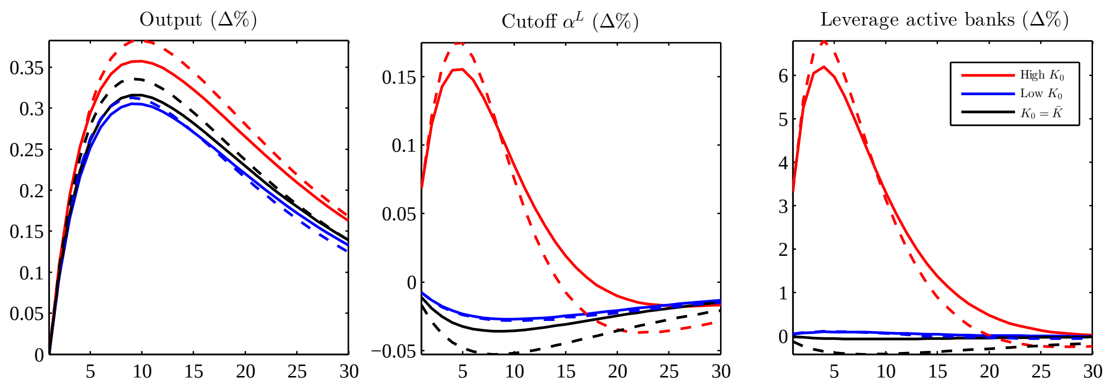

Figure A10: *Monetary policy shock of 100 basis points to γ_t in the model with an interbank market (full lines). For comparison, the baseline model is also plotted (dotted lines).*

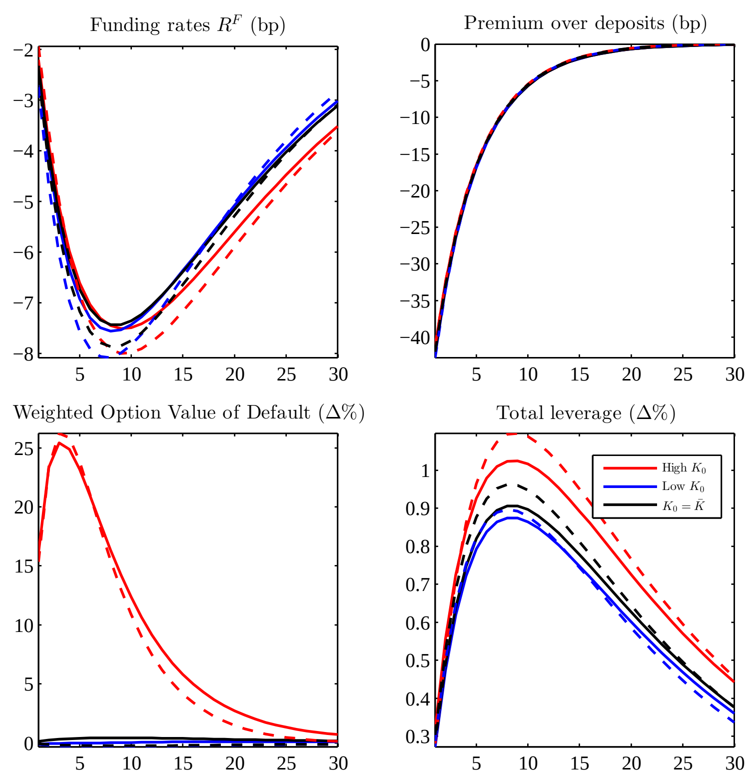

Figure A11: *Monetary policy shock of 100 basis points to γ_t: Financial variables*

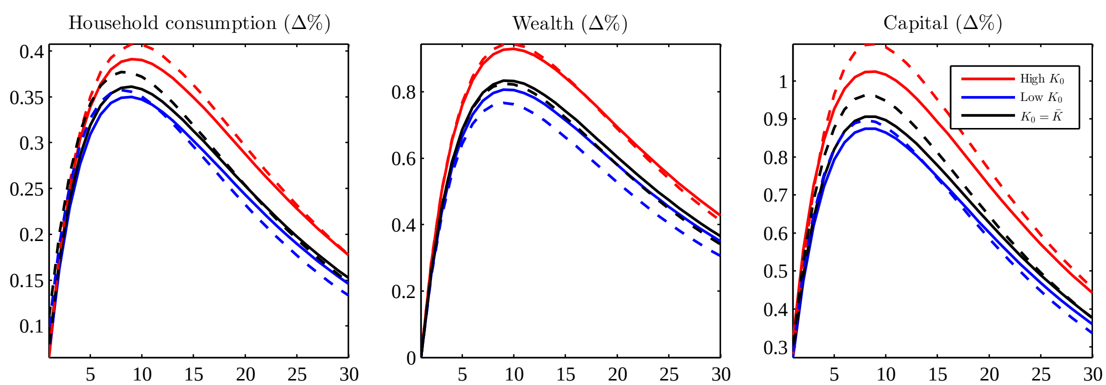

Figure A12: *Monetary policy shock of 100 basis points to γ_t: Real variables*

Appendix F. Comparative statics on size of equity and volatility

Here we explore the role of volatility and net worth in the financial block of the model. We perform two exercises. In the first one we change the parameter σ_z, governing the exogenous volatility of the TFP process. As we can see in figure A13, the main change is in the composition of the financial sector. When volatility is higher, the VaR is tighter and therefore the intensive margin is reduced. Leverage from active intermediaries is lower, which leads to both lower capital stock and cut-off α^L. As it turns out, the lower is volatility, the easier it is for more risk-taking intermediaries to capture more of the market due to the loosening of VaR constraints. This leads to higher systemic risk. There is a *volatility paradox*.

We also look at the effect of changing the parameter ω, the endowment of net worth received by intermediaries. As can be seen in Figure A14, the effect is almost purely compositional with almost no effect on the total amount of capital (differences too marginal to show up in the graph). Given that the right hand side of equation (21) is independent of ω, then changing net worth is just allowing the more risk-taking intermediaries to acquire more assets (given aggregate variables). As with lower volatility, the higher ω is the easier it is for more risk-taking intermediaries to capture a larger share of the market.

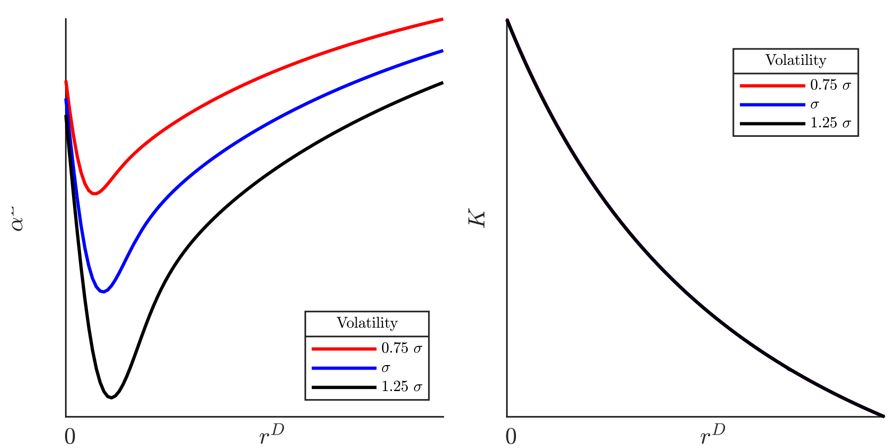

Figure A13: Comparative statics on volatility and interest rates for the financial sector block

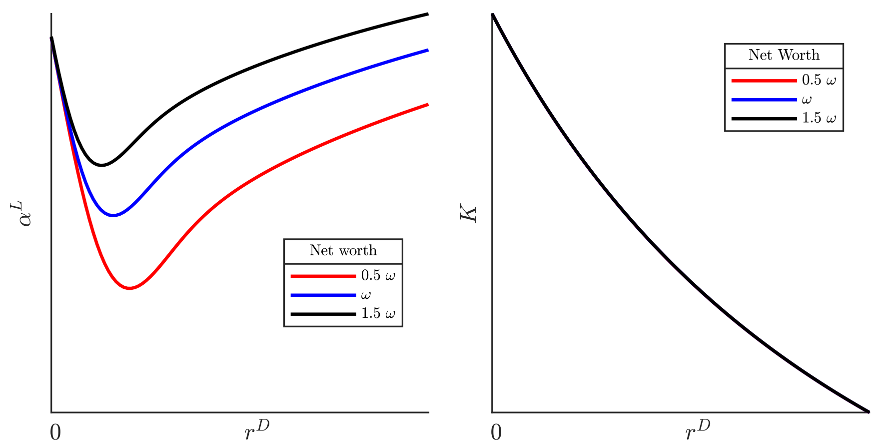

Figure A14: Comparative statics on net worth and interest rates for the financial sector block