# Comments on "Rethinking Multiple Equilibria in Macroeconomic Modelling" by Morris and Shin

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It is a real pleasure to comment on a paper which is of great interest, addresses a fundamental issue in macroeconomics and is also very elegant.

In a series of articles, Steve Morris and Hyun Song Shin have developed a fruitful line of research that extends and applies sophisticated game-theoretic concepts to traditional macroeconomic problems. In this paper, which may be seen to some degree as a synthesis of their approach, they use a simple bank run model as a framework to ask a very important question: Are multiple equilibria in economics the unintended consequence of too simplistic assumptions?

The answer provided by the paper is unambiguously yes. The authors write, for example: "We doubt that economic agents' beliefs are as indeterminate as implied by the multiple equilibrium models. Instead, the apparent indeterminacy of beliefs can be seen as the consequence of two modelling assumptions introduced to simplify the theory. First, the economic fundamentals are assumed to be common knowledge; and second, economic agents are assumed to be certain about other's behavior in equilibrium." The paper then claims that introducing a small amount of idiosyncratic uncertainty is enough to destroy the perfect coordination of agents' actions and beliefs and therefore to eliminate the possibility of multiple equilibria. Since our world seems indeed to be one of imperfect and asymmetric information, this realistic generalization of our traditional macroeconomic models appears to banish multiple equilibria once and for all. They become an "artifact of simplifying assumptions that deliver more than they intended to deliver", as the authors put it.

In my discussion, I will emphasize that the Morris-Shin paper does not in fact eliminate the possibility of multiple equilibria. I will also discuss the robustness of their results more precisely and perform some comparative statics exercises. Finally, I will comment on the empirical applicability of their model and its relations to the literature on multiple equilibria.

## 1 Unique equilibrium?

Morris-Shin sets up a Diamond-Dybvig bank run model with a slightly more sophisticated information structure than usual. Returns follow a normal distribution with a given precision $\alpha$; this is public information. On the other hand, each agent gets a signal with precision $\beta$ regarding the realization of the return;

[1]These comments benefited from discussions with Harald Hau, Thomas Philippon, Richard Portes, David Romer and Mike Woodford.

this is private information. When the fundamentals are common knowledge, it is well known that the Diamond-Dybvig model gives rise to multiple equilibria. By introducing a little bit of noise (a very small degree of asymmetric information), the authors show that the equilibrium is unique. So a very minor modification to an otherwise standard model is able to eliminate the multiplicity of equilibria.

This is a very strong result. I will argue, however, that the minor deviation from the benchmark model chosen by the authors brings with it a lot of interesting and sometimes puzzling results, some of them not emphasized in the paper. In particular, if one does not look exclusively at the limiting case on which the authors are focussing but at the general case of their own model, the possibility for multiple equilibria reappears very naturally.

In the Morris-Shin paper, the condition characterizing the equilibrium is:

$$\rho^* = \Phi\left(\sqrt{\gamma}\left(\rho^* - \overline{r}\right)\right)$$

where $\rho^*$ is the cut-off point below which patient consumers withdraw their money from the bank, $\overline{r}$ is the mean of the returns, $\Phi(.)$ is the cumulative normal distribution and $\gamma$ is a constant given below. Graphically, this equilibrium is illustrated in figure 2.1 of the paper. It is immediately apparent that the 45 degree line and the cumulative normal distribution will intersect only once if the slope of the cumulative normal is "not too steep". Formally a sufficient condition for this to happen is

$$\gamma = \frac{\alpha^2\left(\alpha + \beta\right)}{\beta\left(\alpha + 2\beta\right)} \leq 2\pi$$

When the precision of the private information $\beta$ is very high ($\beta$ goes to infinity for a given $\alpha$, meaning that $\gamma$ becomes very small), the authors interpret their model as being a very small deviation from the standard Diamond-Dybvig model with common knowledge. In that case the Morris-Shin model gives the discontinuity result emphasized in the paper: If private information is very precise, then the two curves intersect only once and we have a unique equilibrium. If, on the other hand, private information is infinitely precise, then we are in the standard Diamond-Dybvig case and there are multiple equilibria. This is an interesting and surprising result and the authors present it very well in the paper.

But this is not the end of the story. Note that there are two different ways to approach the common knowledge case from within the Morris-Shin framework.

[Figure 1 here]

We can approach common knowledge either by letting the precision of the private signal go to infinity, as in the paper, or alternatively we could let the precision of the public information go to infinity. In that latter case, $\alpha$ would be going to infinity for a given $\beta$ and the slope of the cumulative normal distribution would become "very steep" as in figure 2. In that case, there can be multiple

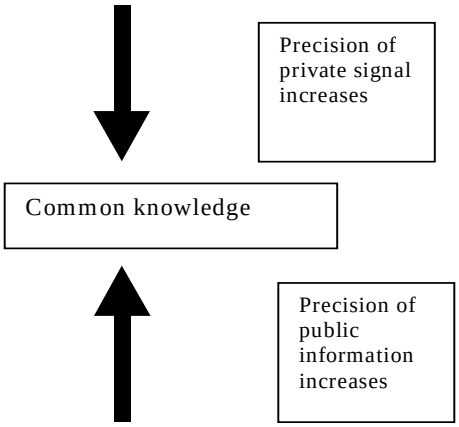


Figure 1: Common knowledge as a limit

equilibria. More generally, it is obvious that as long as the precision of the public information is relatively high compared to the precision of the private signal, multiple equilibria will still exist. This result is intuitive: the more precise public information is, the closer we are to the standard case of common knowledge among economic agents, which is known to generate multiple equilibria. At another level, it is however somehow paradoxical to think that the economies that are generating the more accurate publicly available information are also the ones which are the more prone to multiple equilibria. And, conversely, it is also puzzling that for a given degree of precision of the private signals, economies with very diffuse public information will converge to a unique equilibrium.

[Figure 2 here]

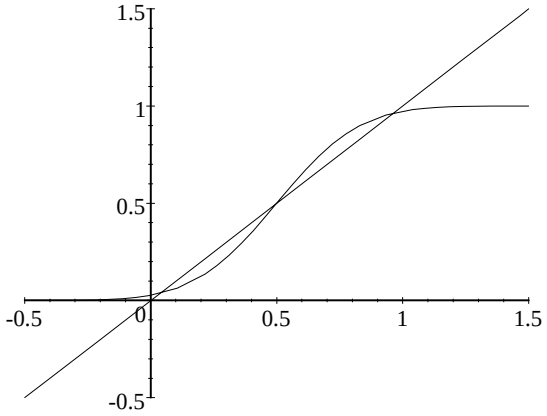


Figure 2: Multiple Equilibria in Morris-Shin

To summarize, the central claim of the paper (the discontinuity result), that a very minor deviation from the standard models with common knowledge is enough to eliminate the possibility of multiple equilibria, is not the whole story: if one does accept that the Morris-Shin framework is a better representation of reality, one has to recognize that this model also delivers multiple equilibria for some parameter domains. Futhermore, common knowledge can be seen as a limiting case in two different ways: in one case, one converges towards common knowledge with a unique equilibrium, in the other case, one converges towards common knowledge with multiple equilibria.

# 2 Comparative statics results and dynamics

If we limit ourselves to the parameter region where uniqueness of equilibrium prevails, we can perform comparative statics exercises, which paves the way towards policy recommendations. A first thing to look at is the impact of the precision of private and public information on the cut-off signal $x^*$ below which patient depositors will withdraw their money out of the bank. It turns out that an increase in the precision of either type of information may lower or raise the value of the critical signal for a given mean return. This result is puzzling.

Another interesting exercise is to look at the impact of a change in public information versus the impact of a change in private information. One can even characterise by how much a private signal should change to balance the impact of a change in public information so that the strategies of the agents are kept unchanged. Since one of the key aspects of public information in the Morris-Shin paper is that it coordinates the expectations of agents, one would expect that public news would have a relatively bigger effect than private news. This intuition is correct provided one is able to control for the relative precision of the private and public informations, which requires knowing the magnitudes of $\alpha$ and $\beta$.

The model presented is a one-shot game (a repetition of one-shot games in the second part of the paper). It would obviously be very nice to do a dynamic extension of the framework. Careful thought should then be given to the process of information revelation. Let us imagine that economic agents play the game presented in the paper at date t. At date t+1, they will have observed the number of people having run on the bank at date t, which is given by:

$$\ell(r) = \Phi\left(\frac{\sqrt{\beta}}{\alpha}\left(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \overline{r})\right)\right)$$

As soon as the proportion of people withdrawing money is observed, the realized return becomes common knowledge, since all the other parameters are known. If there is some persistence in the return variable, then the precision of the public information is increasing over time ($\alpha$ is increasing for a given $\beta$),and we may exit the unique equilibrium region. Extending the model dynamically therefore requires keeping enough "fuzziness" in the public information.

# 3 Empirical applicability

An interesting feature of the Morris-Shin approach is the ability to address policy issues, thanks to the comparative statics exercises performed around the unique equilibrium. For practical purposes, it is therefore very important to know whether the economy is in a unique or a multiple equilibria region, which depends on the value of the parameter $\gamma$. Since $\gamma$ is not homogeneous in $\alpha$ and $\beta$, figuring out the relative precision of the two types of information is not enough. It matters whether $\alpha$ is 17 rather than 13 or whether $\beta$ is 9 rather than 10. Moreover, just like the number of equilibria themselves, we have seen that the comparative statics results depend on the absolute magnitude of the precision of the public and the private informations. Unfortunately it seems extremely difficult to get an idea of what these numbers are in reality. They partly depend on the interpretation one has of the model itself. Should we think of the private information element of the model as differences in psychology across individuals, so that traders reading the same economic news may form different views on the economy depending on their temperament? Or should we think of it as the degree of precision of "inside information"?

This aspect put aside, we should ask ourselves whether the Morris-Shin approach has empirical implications which can clearly be distinguished from those of the models exhibiting multiple equilibria. The authors argue that their model provides testable implications since it suggests a correlation between fundamentals and outcomes, unlike multiple equilibria models, where the shift from one equilibrium to the other may be due to pure sunspots. This point is interesting. Note, however, that multiple equilibria models also provide some correlations between fundamentals and outcomes. In a self-fulfilling speculative attack model, for example, the parameter space is divided in three regions: one where the fundamentals are so good that there can be no attack, one where the fundamentals are so bad that there is an attack for sure, and an intermediate region where there are multiple equilibria.

Therefore I would argue that the key empirical implication of the Morris-Shin model is not that fundamentals are correlated with outcomes, nor that multiple equilibria do not exist -as discussed above- but rather that the degree of information aggregation matters. Having recognized this fact, there are nice natural experiments which could be used to test the model. One could for example look at the role of polls or surveys in presence of a situation with strategic complementarities (like foreign exchange traders). One could also study the impact of the introduction of a futures market on the evolution of spot prices, the idea being that prices of futures would aggregate the information of market participants. The difficulty of putting numbers on the precisions of public and private information and therefore of pinning down the exact implications of the model -which vary across parameter regions- will however remain.

# 4 Interpretation of multiple equilibria

The main message of multiple equilibria models may be that even when the fundamentals of the economy are almost the same, outcomes can be very different. The sense of this basic message seems empirically quite relevant. The ERM crisis of 1992 for example has often been given as an example of self-fulfilling speculative attack. By fundamentals we usually mean all the variables describing the economy, like GDP, prices, exchange rates, etc... except the information structure. A great virtue of the Morris-Shin model is that it introduces the information structure into the set of the fundamental variables. The question is then whether the model can deliver the flavour of the multiple equilibria models while keeping the uniqueness of the equilibrium.

The paper shows that for some parameter values, introducing some noise makes the equilibrium unique. In that uniqueness region, small changes in the information structure do change the threshold value below which an attack occurs, but not dramatically so (in general). In figure 2.1 of the paper for example, one can see that a small change in the information structure (resp. in the mean return) will change the slope of the normal distribution (resp. shift it). But this will not result in a big variation in $\rho^*$, the posterior belief below which the patient consumers withdraw their money, unless the slope of the normal distribution is quite steep, which is exactly the case when one is close to the region where multiple equilibria exist. In other words, the Morris-Shin model can have the flavour of multiple equilibrium models, but this is provided one is in a parameter region away from the limit case considered by the authors and close to the multiplicity domain.

In the absence of even more sophisticated ways to model information aggregation and the endogeneity of the information structure, we are still left with a multiple equilibrium region where we cannot say much about equilibrium selection. Perhaps a phenomenon like the 1992 ERM crisis could be modelled as unique equilibrium once dramatic shifts in information aggregation are incorporated explicitly. One way forward could be to think harder about the information aggregation process: here private information is costless to acquire and is automatically given to all agents. Costly and voluntary information acquisition should ideally be related to the other fundamentals of the economy.

# 5 Conclusions

The paper makes a very important contribution to the literature on strategic complementarities. First, the Morris-Shin approach can be applied to a wide spectrum of issues. We have many macroeconomic models which exhibit multiple equilibria, whether they are used to discuss bank runs, speculative attacks, industrialization, inflation and poverty traps or thick market externalities. As

the authors point out, this multiplicity is a problem if one wants to perform comparative statics exercises. What is the impact of increasing a tax rate, for example, when the system can switch from one equilibrium to another in a random fashion? Determining the equilibrium to which an economic system will converge is a key issue for policy-makers, and this is where the Morris-Shin approach is so valuable. But as I have pointed out, the Morris-Shin model is not as opposed to the multiple equilibria literature as the authors claim. This is not a criticism, and this underlines that the model has many interesting and rich features, which can be exploited further. The model is not very operational yet as far as empirical tests are concerned, mainly because it is hard to pin down the magnitude of the key parameters which determine the domain of existence of equilibria as well as the comparative statics results. It also lacks true dynamics. The Morris-Shin framework has however already been (rightly) very influential in the way we think about coordination and information aggregation in macroeconomic models and will certainly generate a lot of interesting new results in very diverse areas.